Search-based Planning with Motion Primitives

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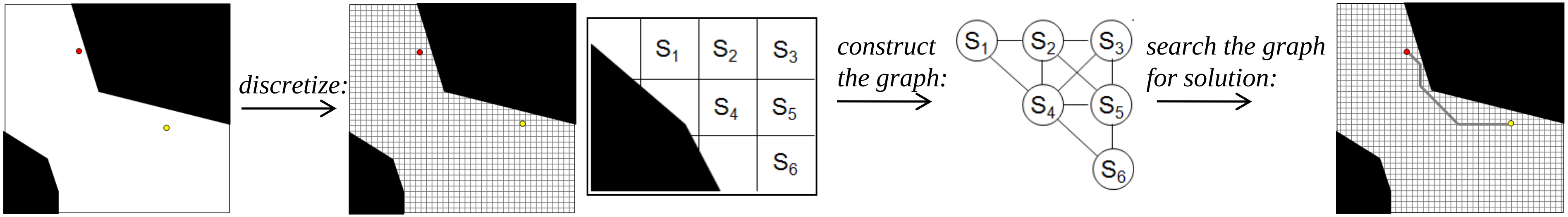
What is Search-based Planning

- generate a graph representation of the planning problem
- search the graph for a solution
- can interleave the construction of the representation with the search (i.e., construct only what is necessary)

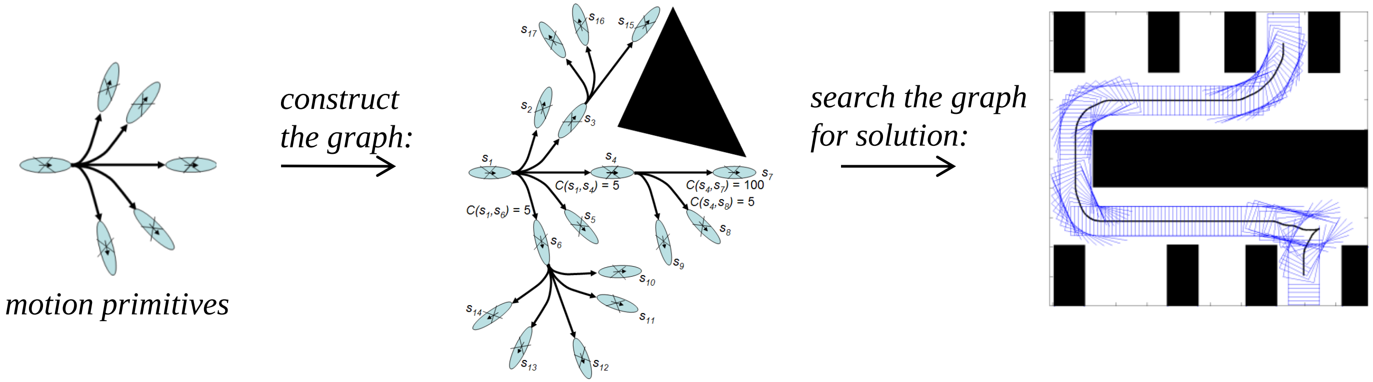
2D grid-based graph representation for 2D (*x*,*y*) search-based planning:


discretize:

construct the graph:

search the graph for solution:

lattice-based graph representation for 3D (*x*,*y*,*θ*) planning:

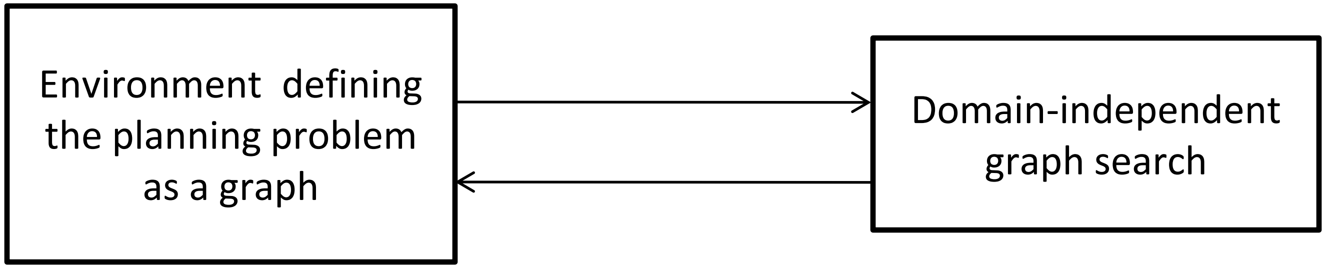

motion primitives

construct the graph:

search the graph for solution:

Search-based Planning Library (SBPL)

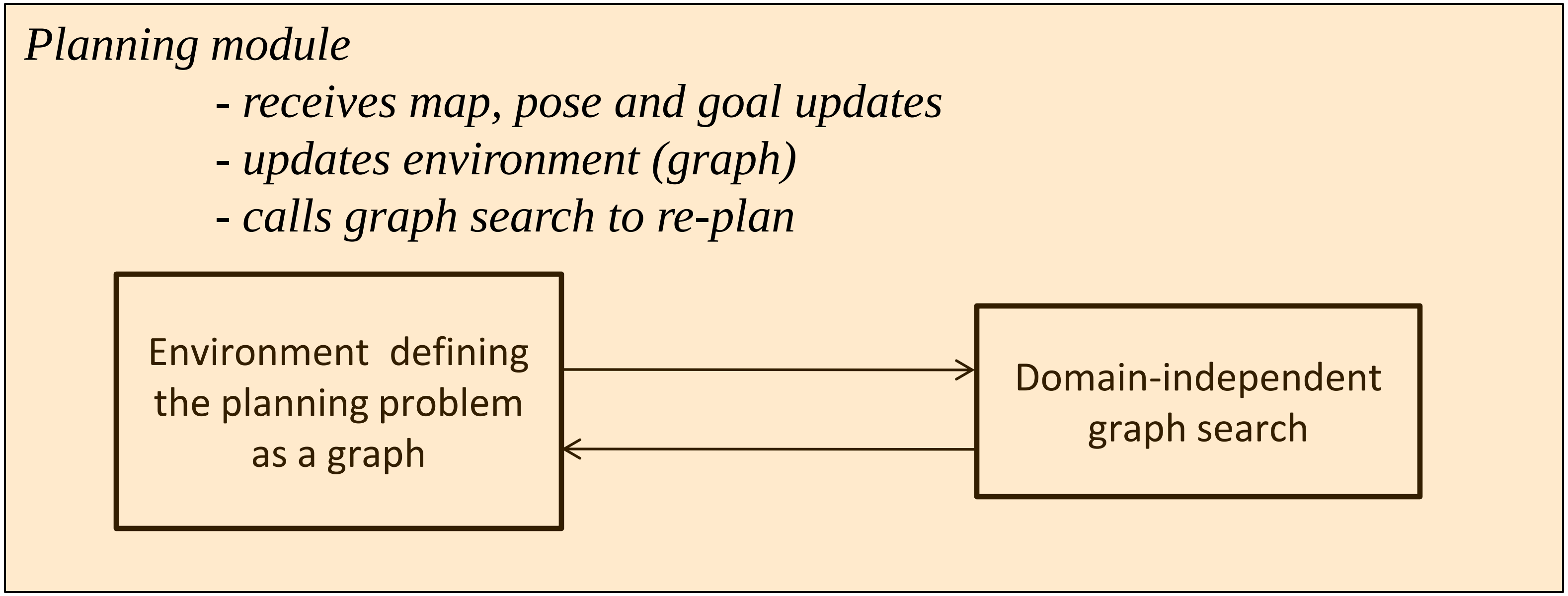
- http://www.ros.org/wiki/sbpl
- SBPL is:
  - a library of domain-independent graph searches
  - a library of environments (planning problems) that represent the problems as graph search problems
  - designed to be so that the same graph searches can be used to solve a variety of environments (graph searches and environments are independent of each other)
  - a standalone library that can be used with or without ROS and under linux or windows



Search-based Planning Library (SBPL)

- http://www.ros.org/wiki/sbpl

- SBPL can be used to:
  - implement particular planning modules such as x,y,θ planning and arm motion planning modules within ROS
  - design and drop-in new environments (planning problems) that represent the problem as a graph search and can therefore use existing graph searches to solve them
  - design and drop-in new graph searches and test their performance on existing environments



Planning module

- receives map, pose and goal updates

- updates environment (graph)

- calls graph search to re-plan

Search-based Planning Library (SBPL)

- Currently implemented graph searches within SBPL:
    - ARA* - anytime version of A*
    - Anytime D* - anytime incremental version of A*
    - R* - a randomized version of A* (hybrid between deterministic searches and sampling-based planning)

- Currently implemented environments (planning problems) within SBPL:
    - 2D (x,y) grid-based planning problem
    - 3D (x,y,θ) lattice-based planning problem
    - 3D (x,y,θ) lattice-based planning problem with 3D (x,y,z) collision checking
    - N-DOF planar robot arm planning problem

- ROS packages that use SBPL:
    - SBPL lattice global planner for (x,y,θ) planning for navigation
    - SBPL cart planner for PR2 navigating with a cart
    - SBPL motion planner for PR2 arm motions
    - default move_base invokes SBPL lattice global planner as part of escape behavior

- Unreleased ROS packages and other planning modules that use SBPL:
    - SBPL door planning module for PR2 opening and moving through doors
    - SBPL planning module for navigating in dynamic environments
    - 4D planning module for aerial vehicles (x,y,z,θ)

…

What I will talk about

- Graph representations (implemented as environments for SBPL)
  - 3D (*x*,*y*,θ) lattice-based graph (within SBPL)
  - 3D (*x*,*y*,θ) lattice-based graph for 3D (*x*,*y*,*z*) spaces (within SBPL)
  - Cart planning (separate SBPL-based package)
  - Lattice-based arm motion graph (separate SBPL-based motion planning module)
  - Door opening planning (separate SBPL-based package)

- Graph searches (implemented within SBPL)
  - ARA* - anytime version of A*
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- Heuristic functions (implemented as part of environments)

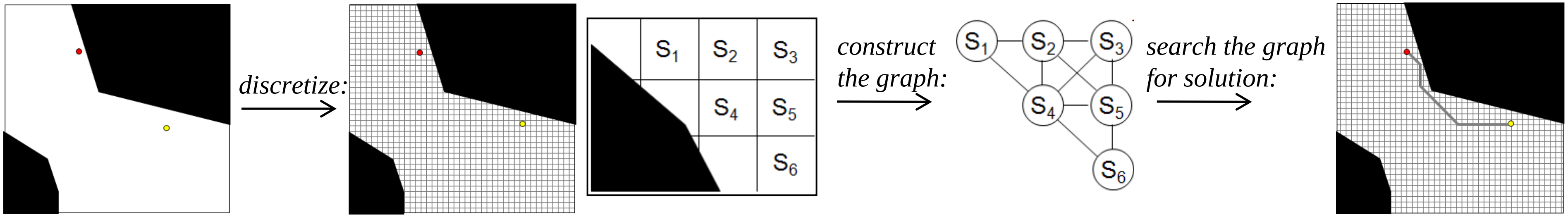
- Overview of how SBPL code is structured

- What's coming

Lattice-based Graphs for Navigation

- Problems with (very popular) pure grid-based planning

2D grid-based graph representation for 2D (*x*,*y*) search-based planning:


discretize:

construct the graph:

search the graph for solution:

sharp turns do not incorporate the kinodynamics constraints of the robot

Lattice-based Graphs for Navigation

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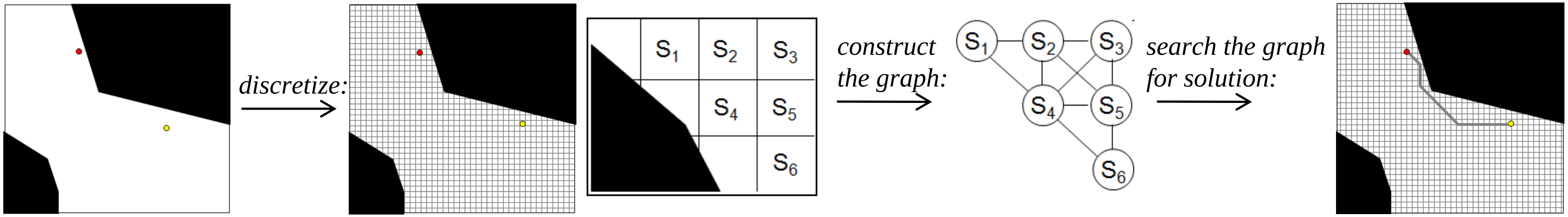


discretize:

construct the graph:

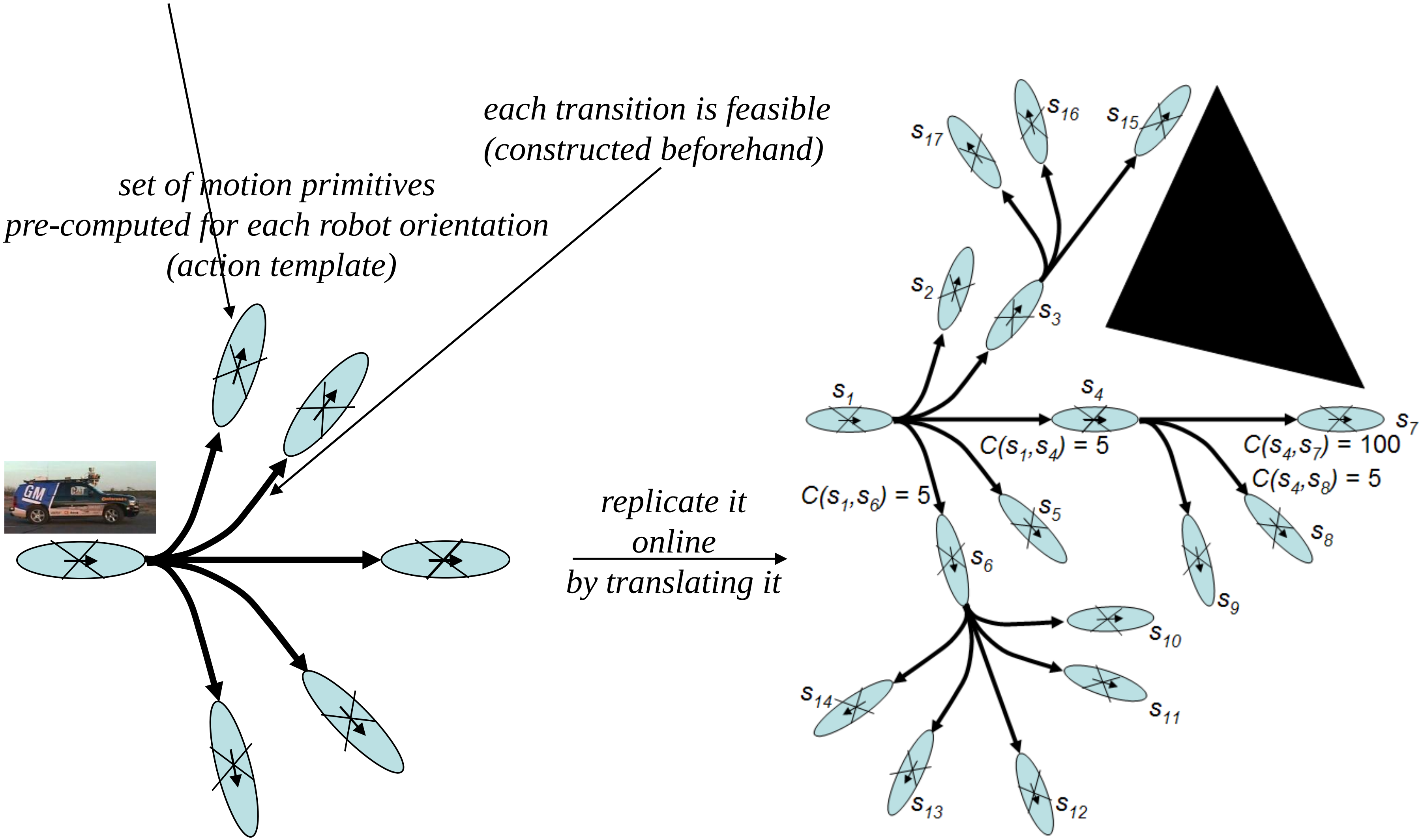
search the graph for solution:

3D-grid (x,y,θ) would help a bit
but won't resolve the issue

Lattice-based Graphs for Navigation

- Graphs constructed using motion primitives [Pivtoraiko & Kelly, ‘05]

outcome state is the center of the corresponding cell in the underlying (x,y,θ,…) cell



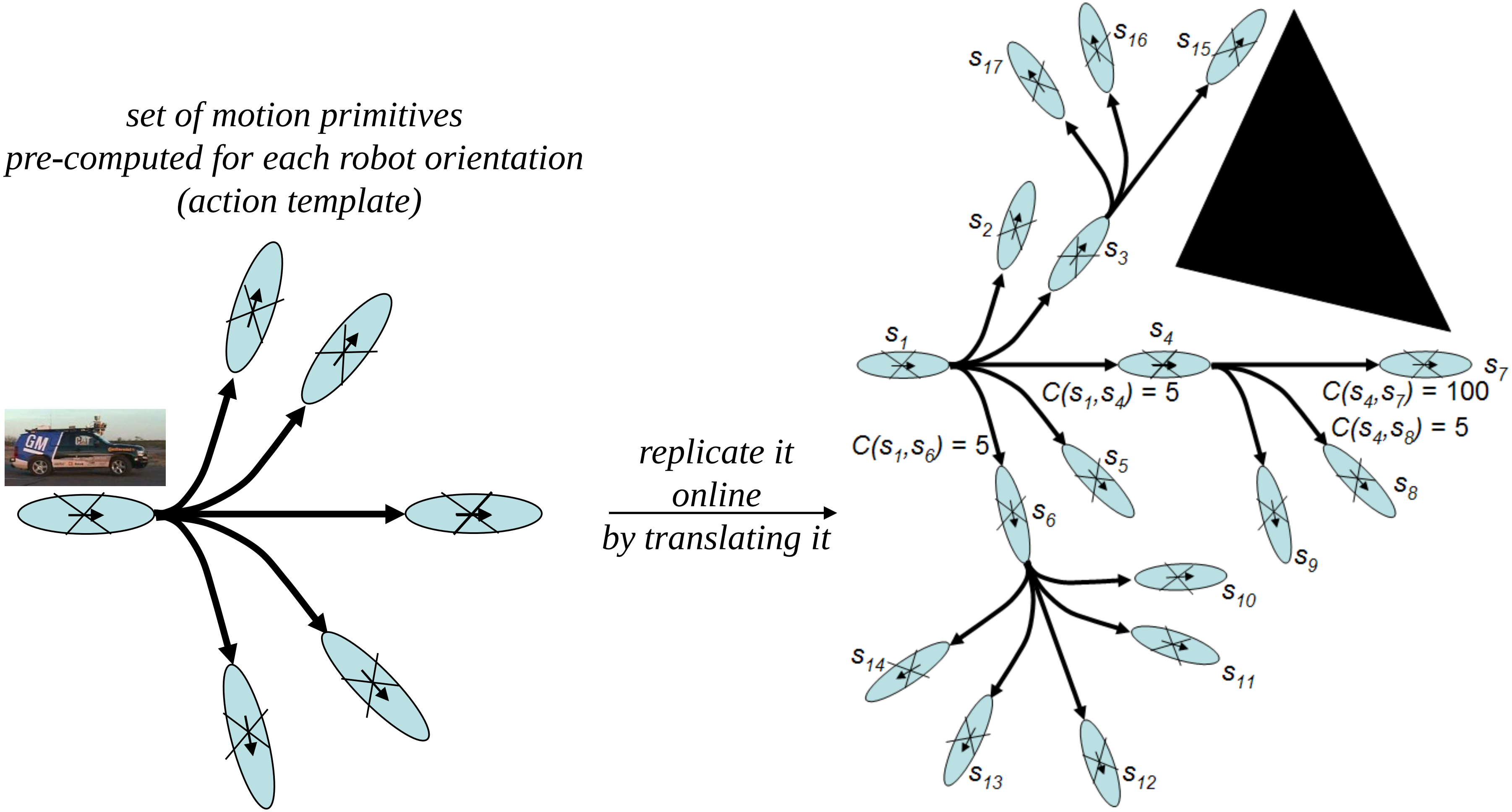
each transition is feasible (constructed beforehand)

set of motion primitives pre-computed for each robot orientation (action template)

replicate it online by translating it

Lattice-based Graphs for Navigation

- Graphs constructed using motion primitives [Pivtoraiko & Kelly, ‘05]
    - pros: sparse graph, feasible paths, can incorporate a variety of constraints
    - cons: possible incompleteness



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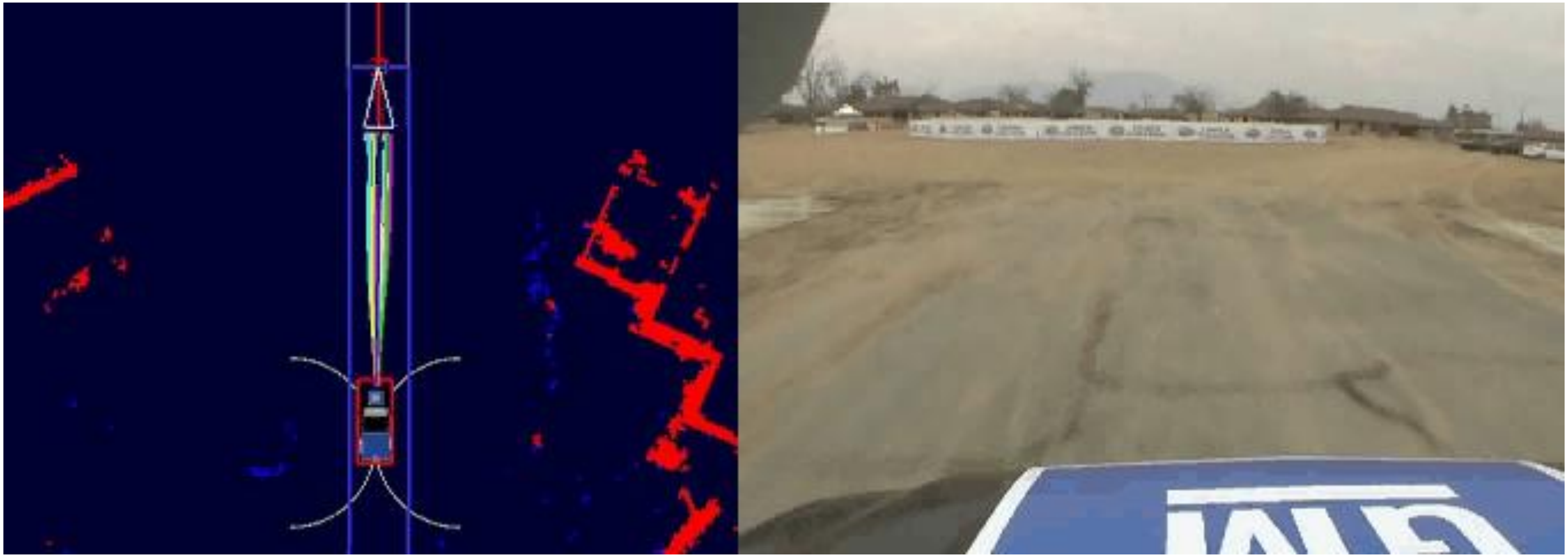
*planning on 4D (<x,y,orientation,velocity>) multi-resolution lattice using Anytime D**
[Likhachev & Ferguson, ‘09]



part of efforts by Tartanracing team from CMU for the Urban Challenge 2007 race

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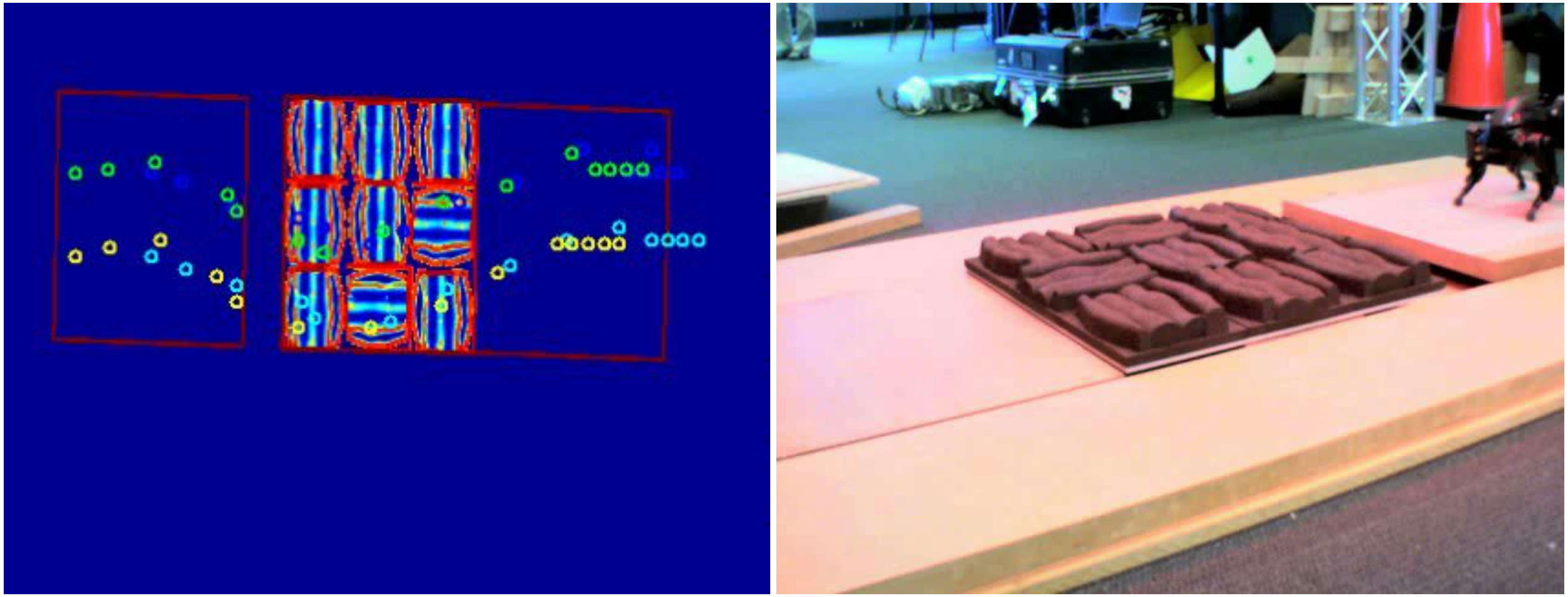
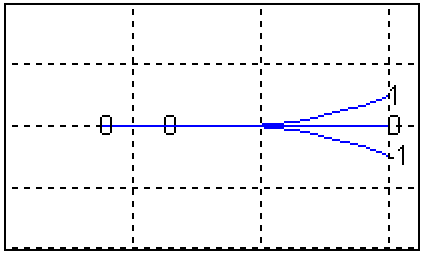


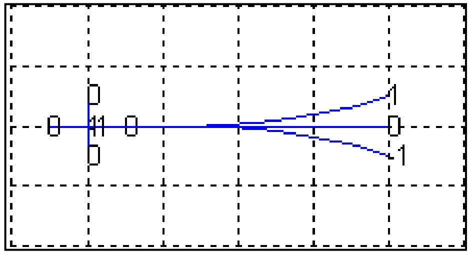
planning in 8D (foothold planning) lattice-based graph for quadrupeds [Vernaza et al.,’09]
using R search [Likhachev & Stentz, ‘08]*

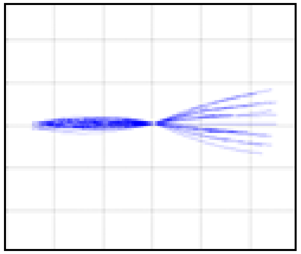
Lattice-based Graphs for Navigation

- 3D (x,y,θ) lattice-based graph representation (*environment_navxythetalat.h/cpp in SBPL*)
  - takes set of motion primitives as input (.mprim files generated within matlab/mprim directory using corresponding matlab scripts):

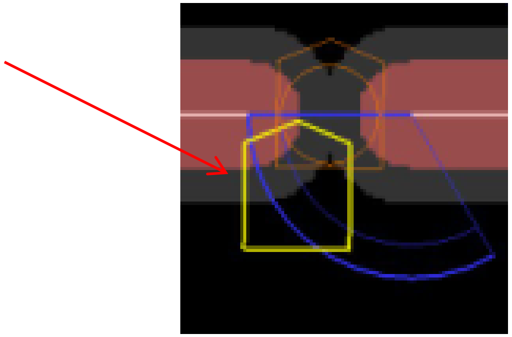
unicycle model *or* *unicycle with sideways motions* *or …*





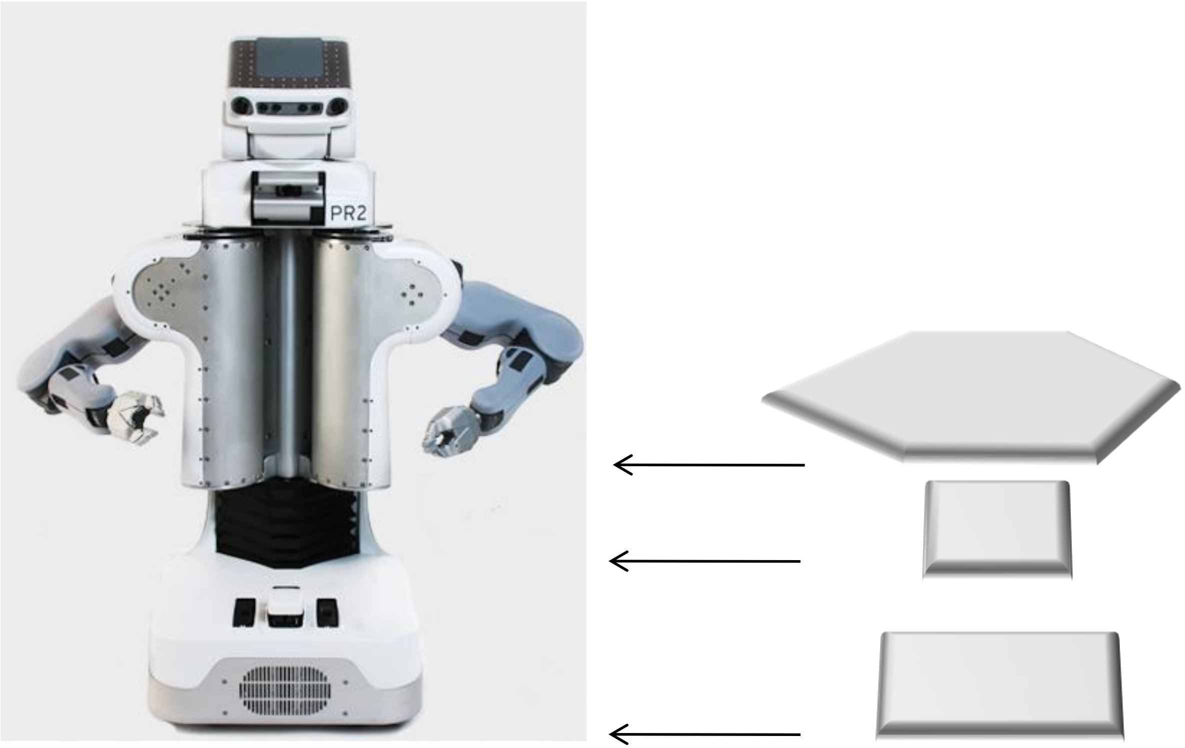


  - takes the footprint of the robot defined as a polygon as input



Lattice-based Graphs for Navigation

- 3D (x,y,θ) lattice-based graph representation for 3D (x,y,z) spaces (*environment_navxythetamlevlat.h/cpp in SBPL*)
  - takes set of motion primitives as input
  - takes *N* footprints of the robot defined as polygons as input.
  - each footprint corresponds to the projection of a part of the body onto *x*,*y* plane.
  - collision checking/cost computation is done for each footprint at the corresponding projection of the 3D map



Graph Representation for Cart Planning

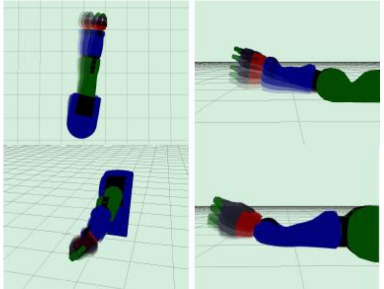
[Scholz, Marthi, Chitta & Likhachev, in submission]

- 3D (x,y,θ,θ_{cart}) lattice-based graph representation (*in a separate Cart Planner package*)
  - takes set of motion primitives *feasible for the coupled robot-cart* system as input (arm motions generated via IK)
  - takes footprints of the robot and the cart defined as polygons as input

Graph Representation for Arm Planning

[Cohen, Chitta & Likhachev, ICRA'10; Cohen et al., in submission]

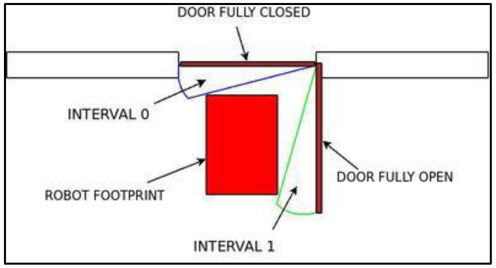
- 7D (*joint angles*) lattice-based graph representation (*in a separate SBPL Arm Planner package*)
  - takes set of motion primitives defining joint angle changes as input →
  - takes joint angle limits and link widths
  - goal is a 6 DoF pose for the end-effector

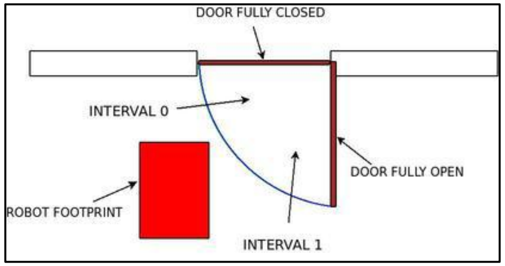


Graph Representation for Door Opening Planning

[Chitta, Cohen & Likhachev, ICRA’10]

- 4D ($x,y,\theta,door\ interval$) graph representation (*in a separate SBPL Door Planner package*)
  - takes set of motion primitives defining feasible $x,y, \theta,door\ angles$ in the door frame as input
  - goal is for the door to be fully open
  - suitable for pushing/pulling doors





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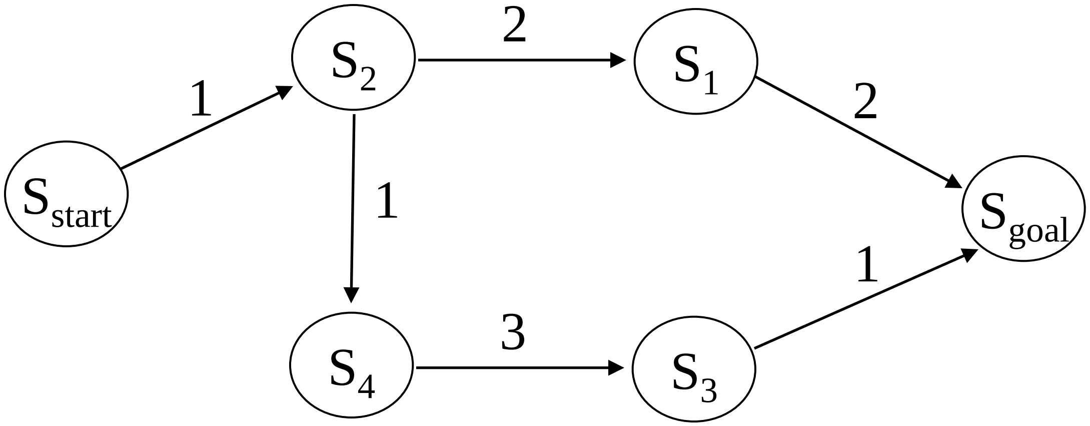
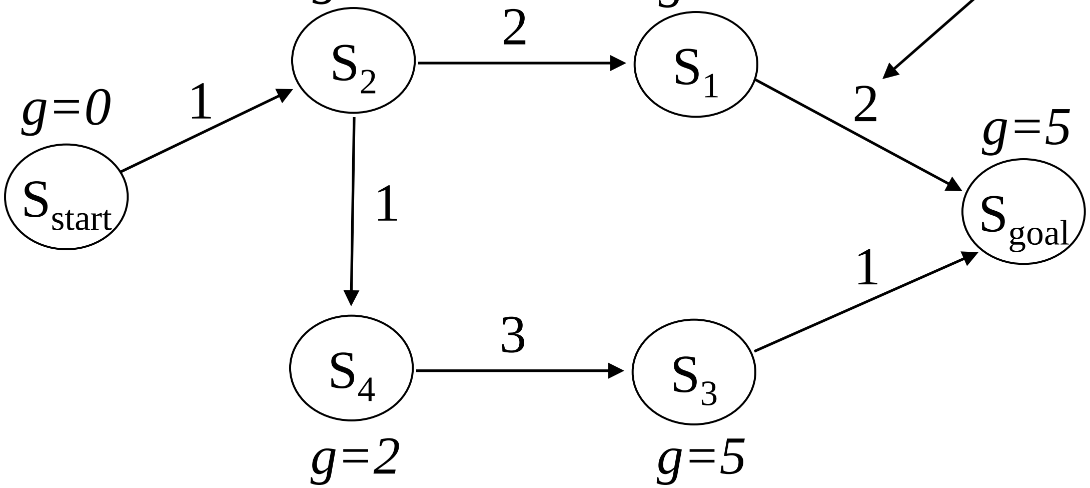
Searching Graphs

- Once a graph is given (defined by environment file in SBPL), we need to search it for a path that minimizes cost as much as possible



Searching Graphs

- Many searches work by computing optimal g-values for relevant states
  - $g(s)$ – an estimate of the cost of a least-cost path from s_{start} to s
  - optimal values satisfy: $g(s) = \min_{s'' \in pred(s)} g(s'') + c(s'',s)$



the cost $c(s_1,s_{goal})$ *of an edge from* s_1 *to* s_{goal}

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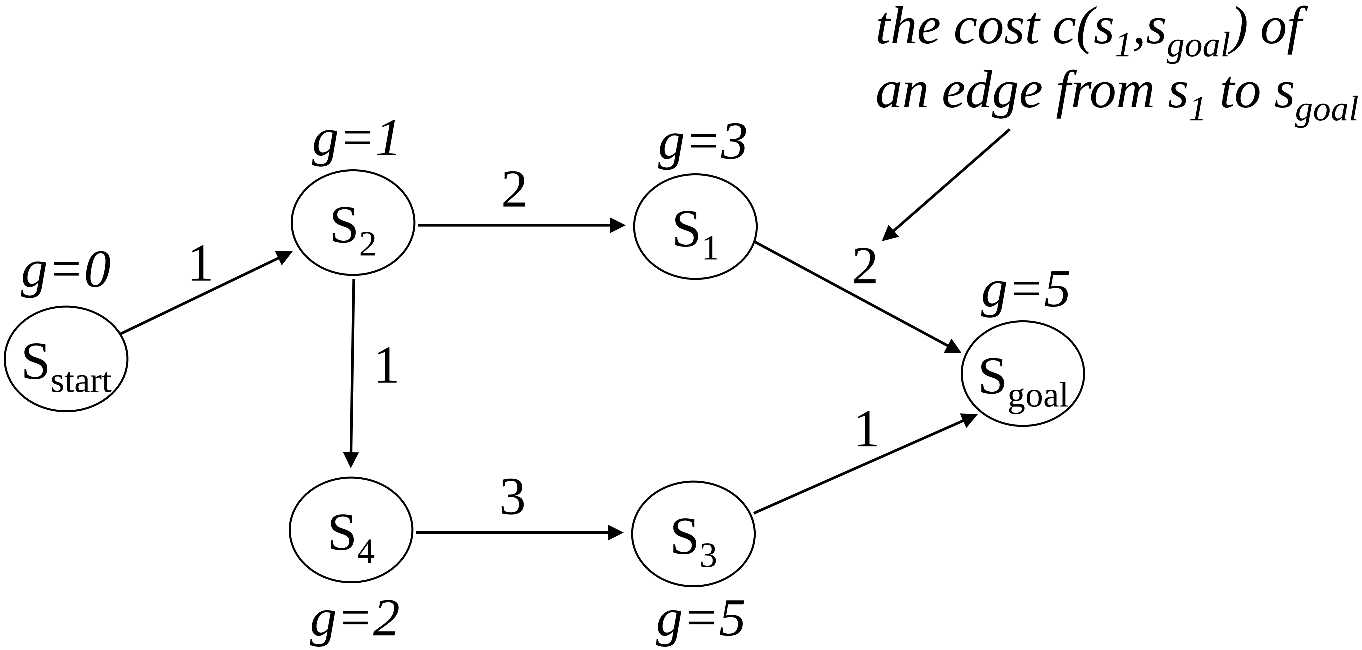
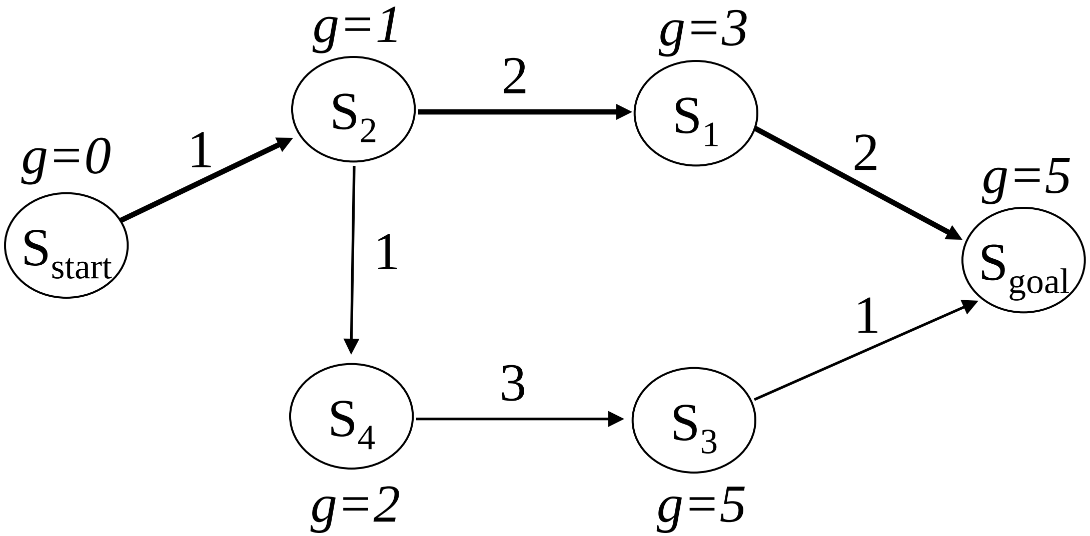


Searching Graphs

- Least-cost path is a greedy path computed by backtracking:
  - start with s_{goal} and from any state s move to the predecessor state s' such that

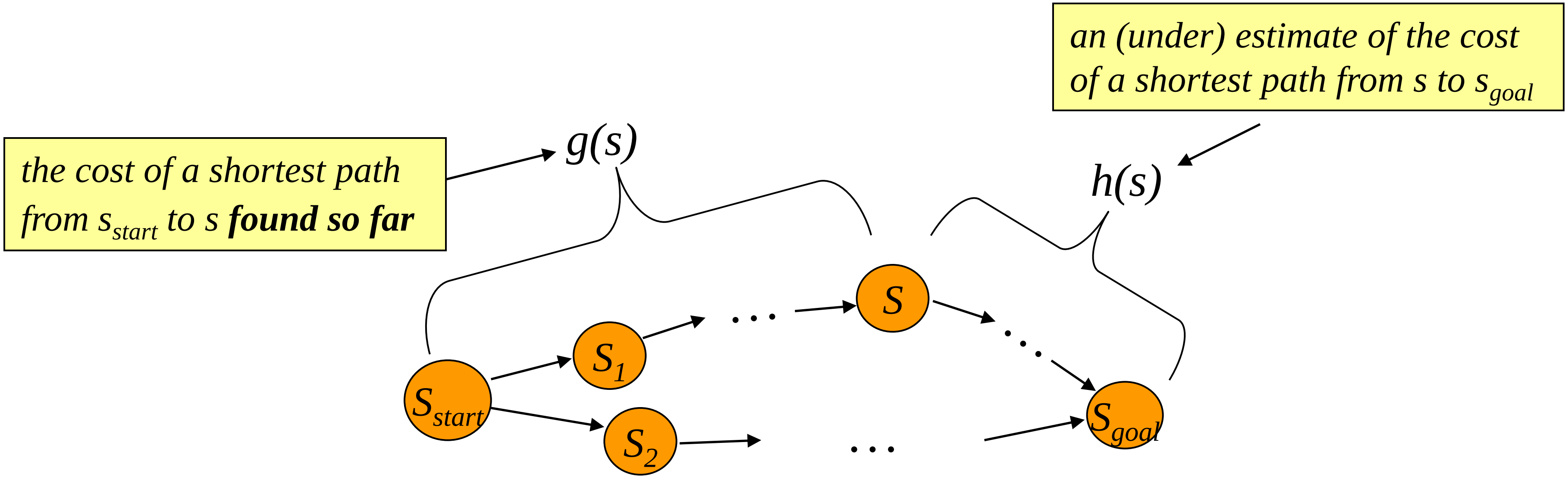
$$s' = \arg\min_{s'' \in pred(s)} (g(s'') + c(s'', s))$$



A* Search

- Computes optimal g-values for relevant states

at any point of time:



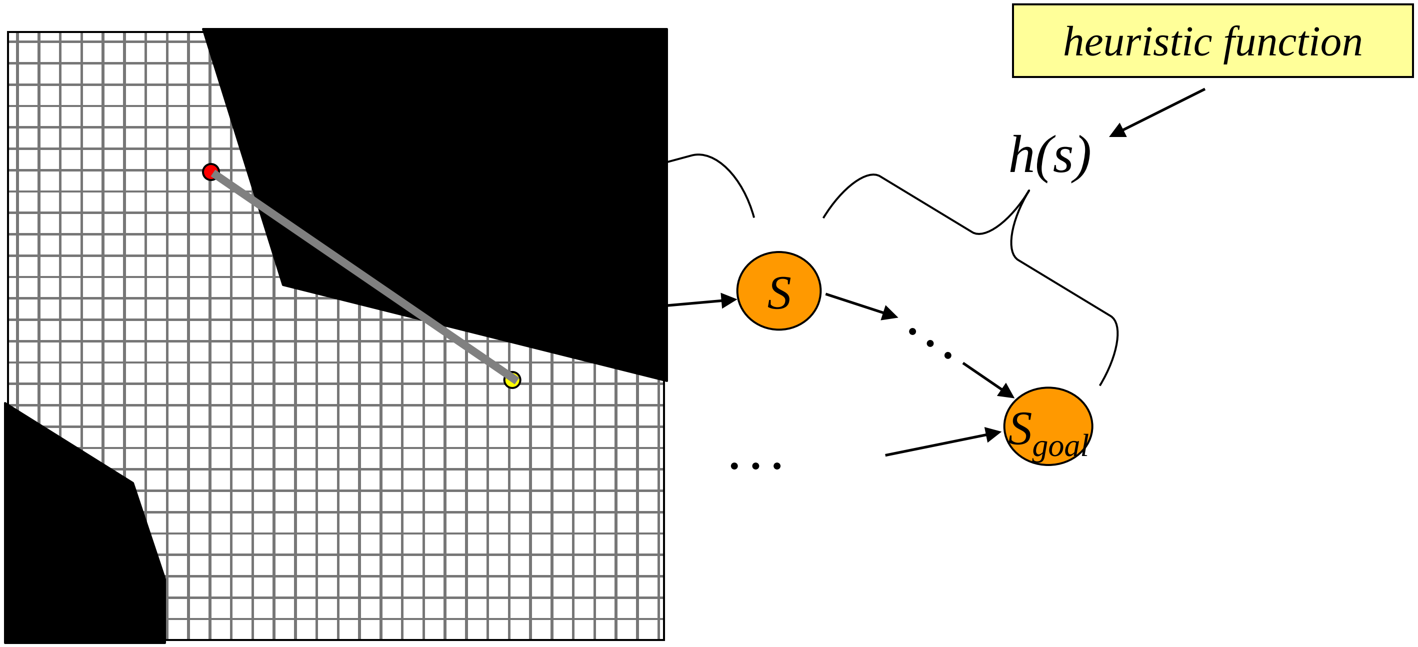
the cost of a shortest path from s_{start} to s ***found so far***

an (under) estimate of the cost of a shortest path from s to s_{goal}

A* Search

- Computes optimal g-values for relevant states

at any point of time:



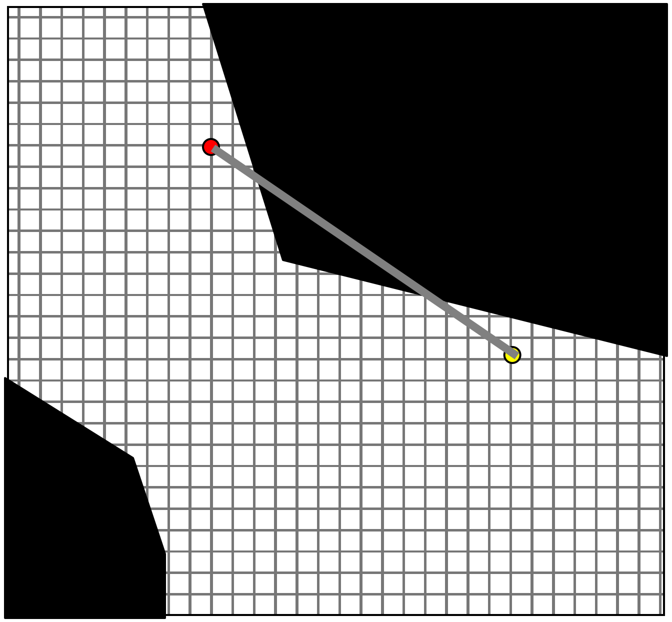
one popular heuristic function – Euclidean distance

A* Search

minimal cost from s to s_{goal}

- Heuristic function must be:
  - admissible: for every state s, $h(s) \leq c^*(s,s_{goal})$
  - consistent (satisfy triangle inequality):

    $h(s_{goal},s_{goal}) = 0$ *and for every* $s \neq s_{goal}$, $h(s) \leq c(s,succ(s)) + h(succ(s))$
  - admissibility follows from consistency and often consistency follows from admissibility



A* Search

- Computes optimal g-values for relevant states

Main function

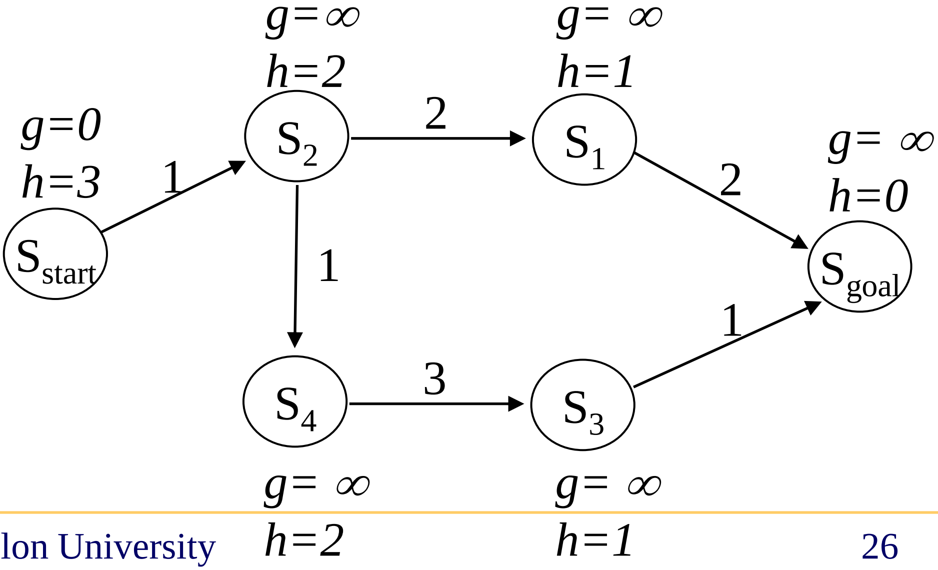
```
$g(s_{start}) = 0$; all other $g$-values are infinite; $OPEN = \{s_{start}\}$;
ComputePath();
publish solution;
```

set of candidates for expansion

ComputePath function

```
while($s_{goal}$ is not expanded)
    remove $s$ with the smallest $[f(s) = g(s)+h(s)]$ from $OPEN$;
    expand $s$;
```

for every expanded state
g(s) is optimal
(if heuristics are consistent)



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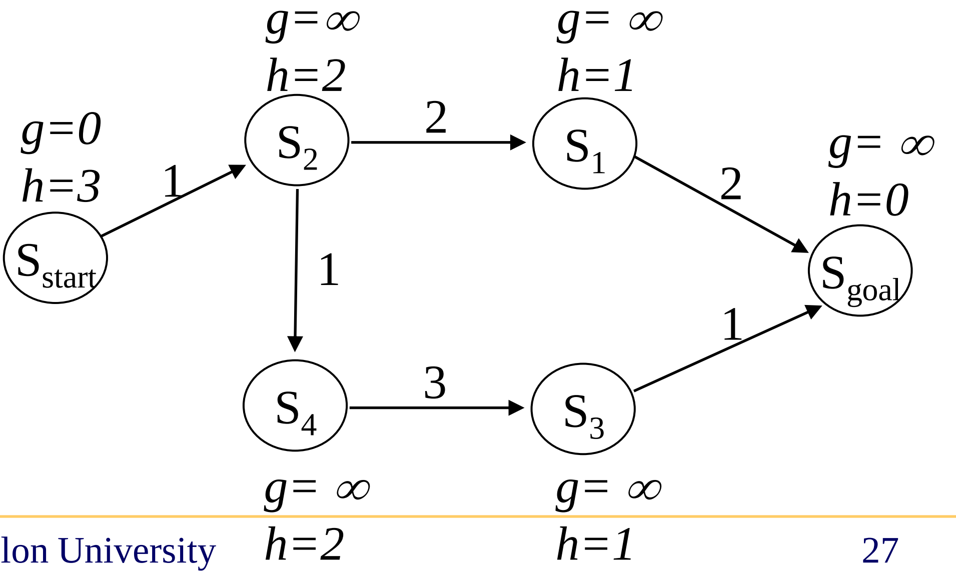

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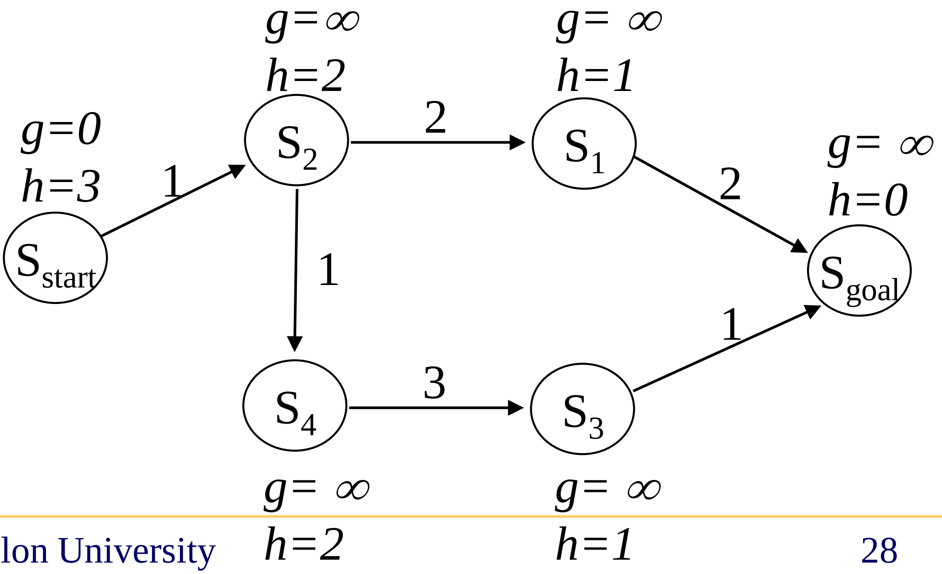
        if $g(s') > g(s) + c(s,s')$
            $g(s') = g(s) + c(s,s')$;
            insert $s'$ into $OPEN$;
```

set of states that have already been expanded

tries to decrease g(s') using the found path from s_{start} to s



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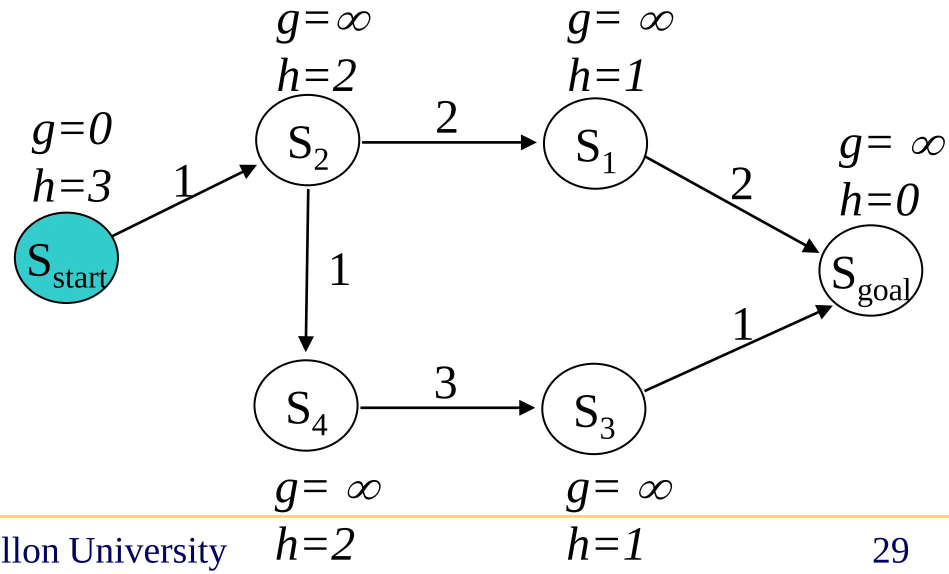
```

CLOSED = {}

OPEN = $\{s_{start}\}$

next state to expand: s_{start}



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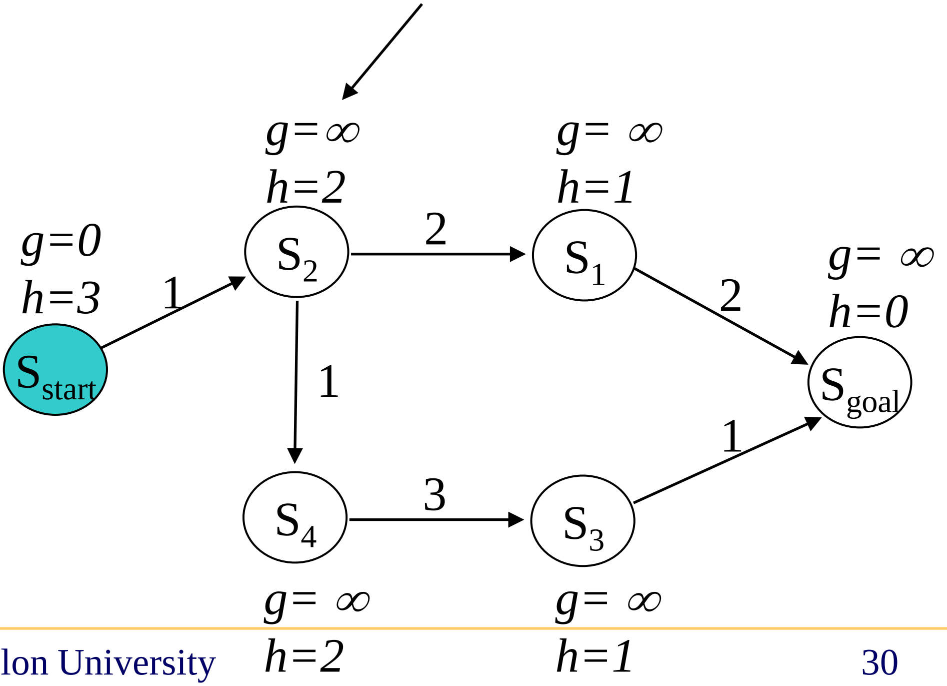
```

$g(s_2) > g(s_{start}) + c(s_{start}, s_2)$



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next state to expand: s_{start}

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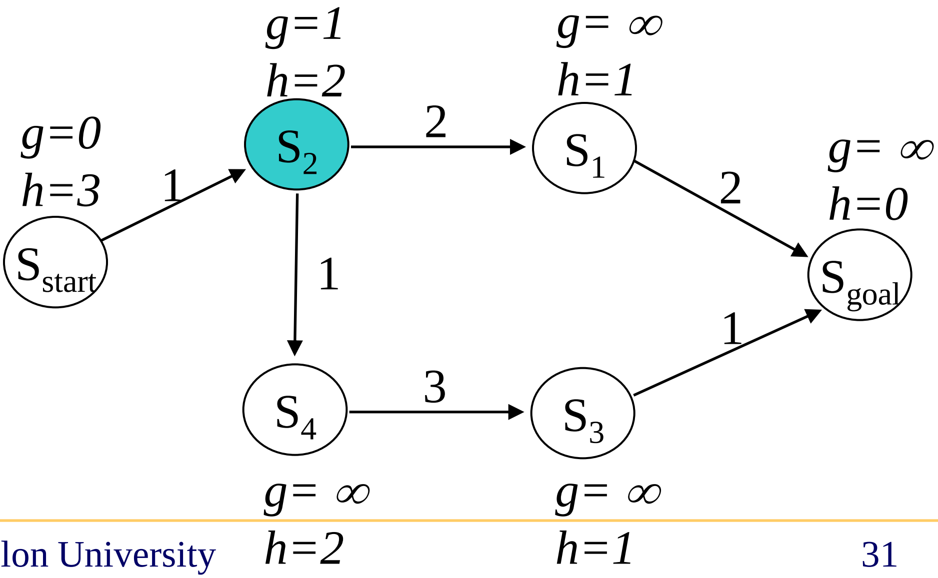

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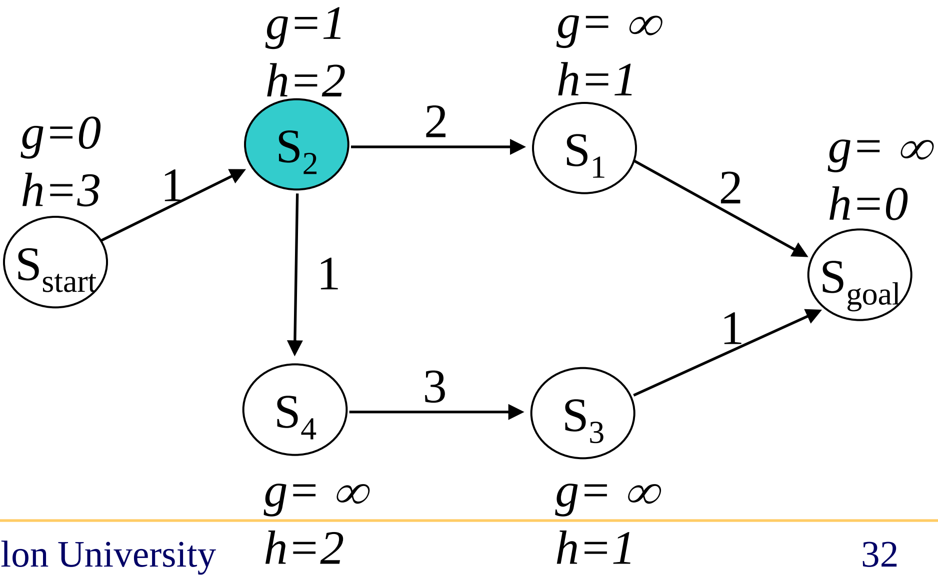
```

$CLOSED = \{s_{start}\}$

$OPEN = \{s_2\}$

next state to expand: s_2



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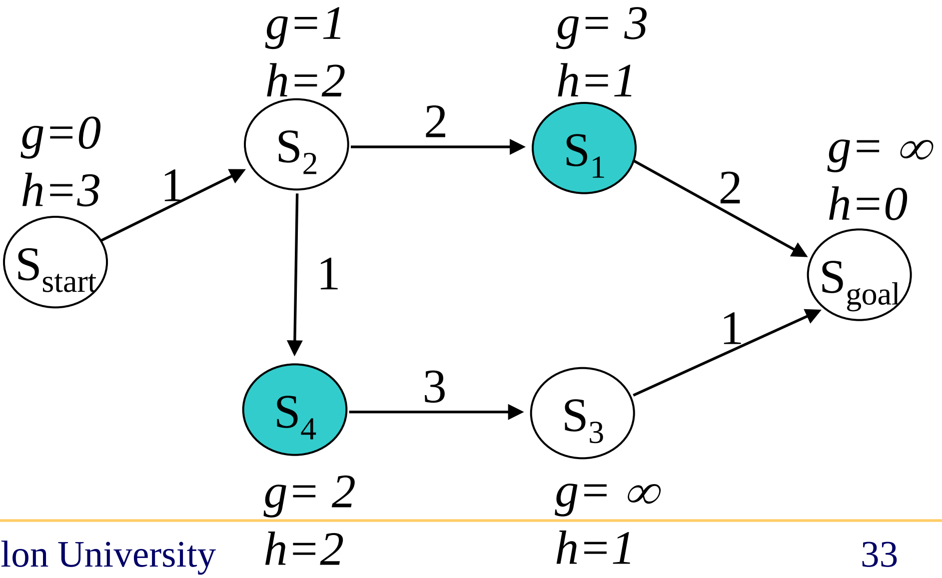
        insert $s'$ into $OPEN$;
```

$CLOSED = \{s_{start}, s_2\}$

$OPEN = \{s_1, s_4\}$

next state to expand: s_1



A* Search

- Computes optimal g-values for relevant states

ComputePath function

```
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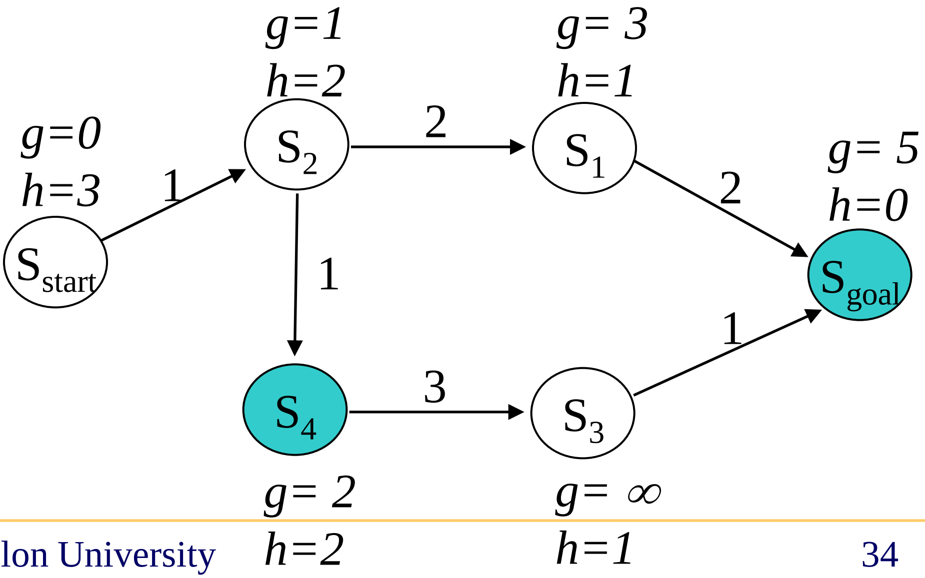
  remove s with the smallest $[f(s) = g(s)+h(s)]$ from OPEN;
  insert s into CLOSED;
  for every successor s' of s such that s' not in CLOSED
      if $g(s') > g(s) + c(s,s')$
        $g(s') = g(s) + c(s,s');$
        insert s' into OPEN;
```

$CLOSED = \{s_{start}, s_2, s_1\}$

$OPEN = \{s_4, s_{goal}\}$

next state to expand: s_4



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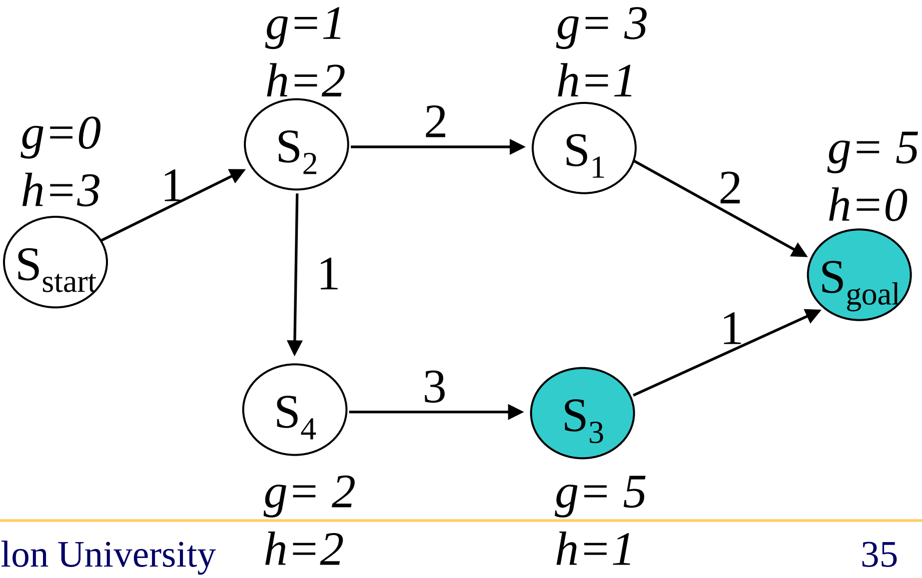
        $g(s') = g(s) + c(s,s')$;
        insert $s'$ into $OPEN$;
```

$CLOSED = \{s_{start}, s_2, s_1, s_4\}$

$OPEN = \{s_3, s_{goal}\}$

next state to expand: s_{goal}



A* Search

- Computes optimal g-values for relevant states

ComputePath function

```
while($s_{goal}$ is not expanded)
   remove $s$ with the smallest $[f(s) = g(s)+h(s)]$ from $OPEN$;
   insert $s$ into $CLOSED$;
   for every successor $s'$ of $s$ such that $s'$ not in $CLOSED$
      if $g(s') > g(s) + c(s,s')$
         $g(s') = g(s) + c(s,s')$;
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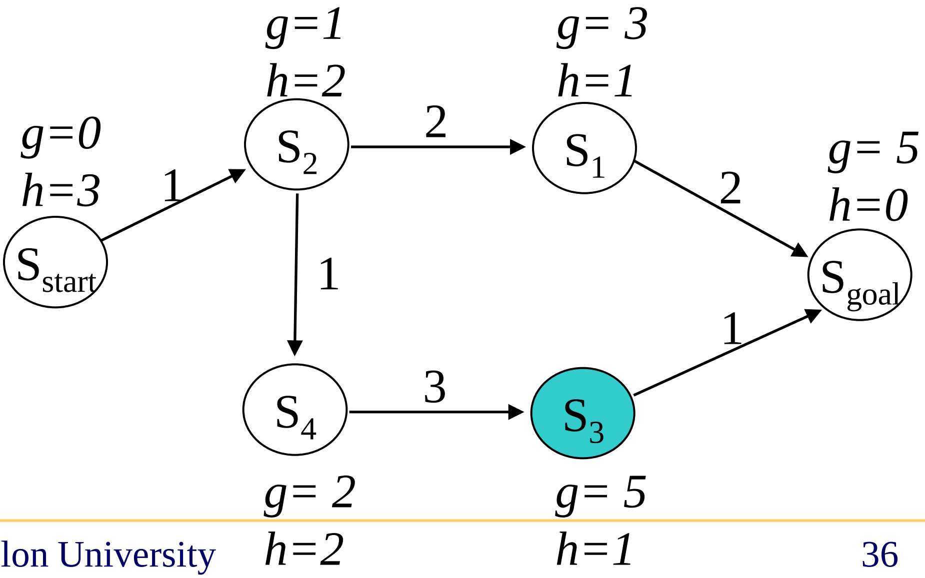
```

$CLOSED = \{s_{start}, s_2, s_1, s_4, s_{goal}\}$

$OPEN = \{s_3\}$

done



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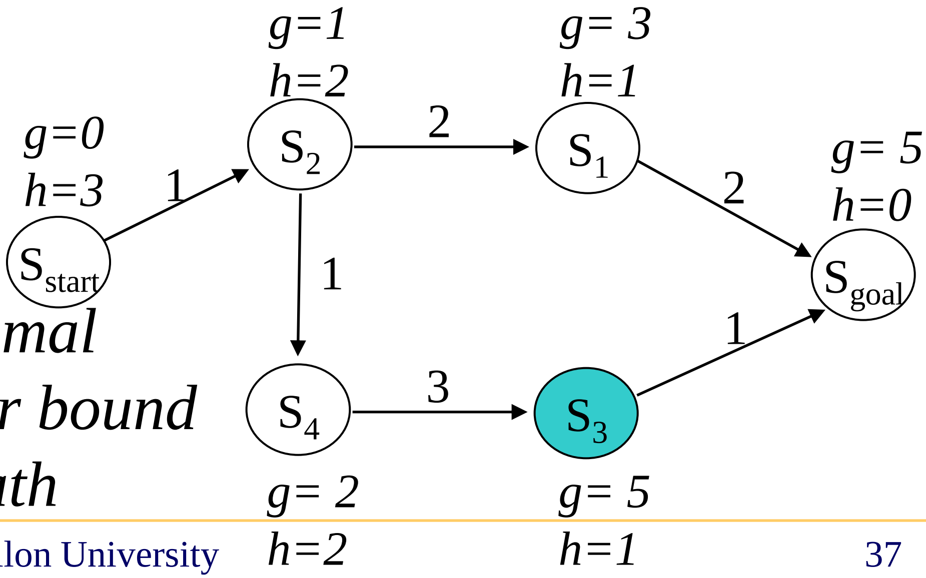

for every expanded state g(s) is optimal
for every other state g(s) is an upper bound
we can now compute a least-cost path

A* Search

- Computes optimal g-values for relevant states

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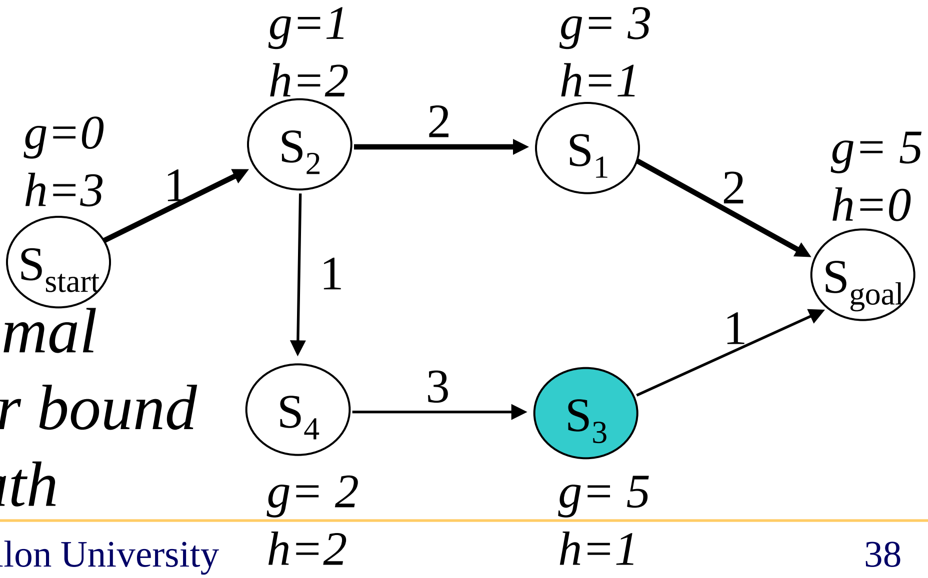

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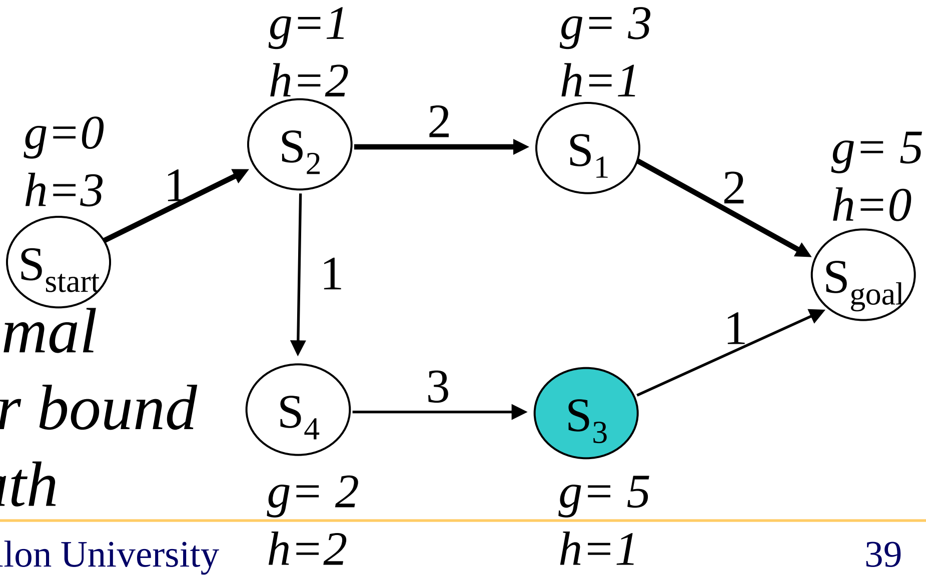
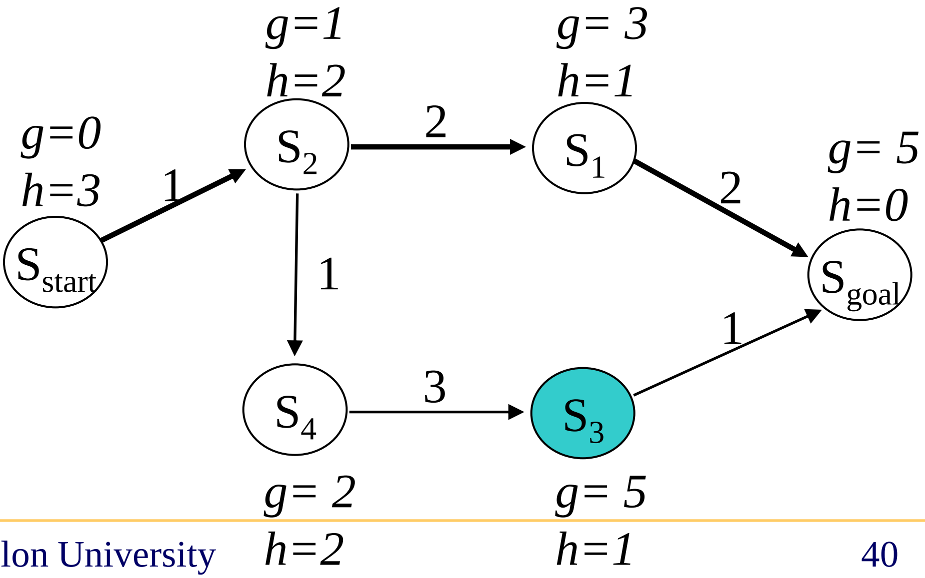


for every expanded state g(s) is optimal
for every other state g(s) is an upper bound
we can now compute a least-cost path

A* Search

- Is guaranteed to return an optimal path (in fact, for every expanded state) – optimal in terms of the solution

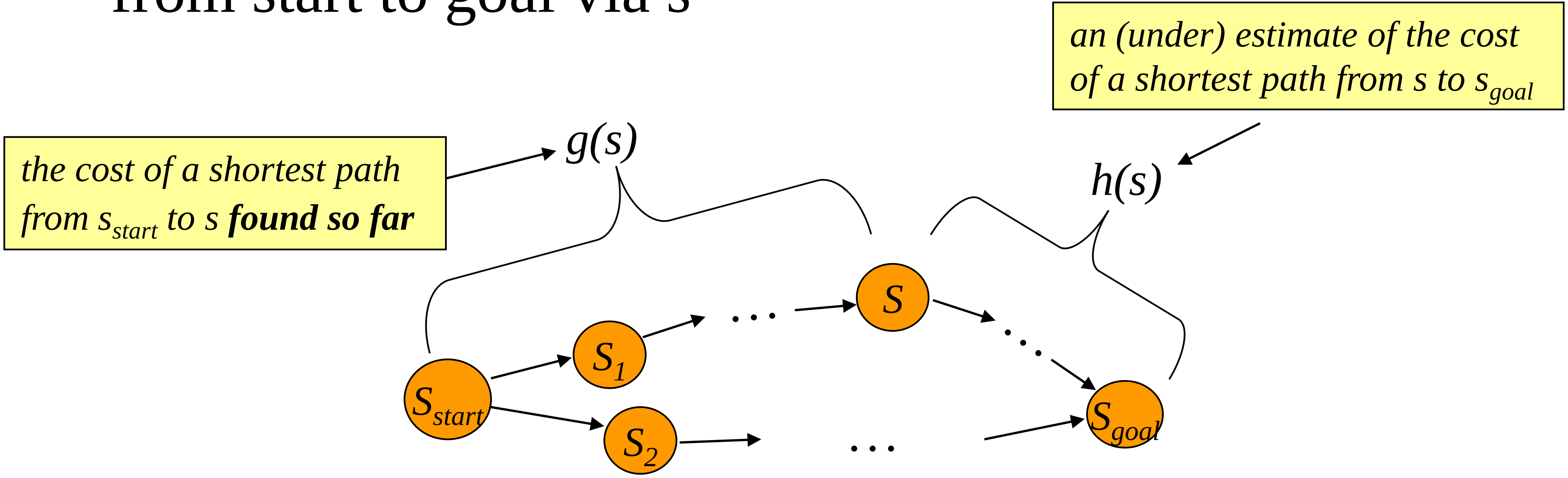
- Performs provably minimal number of state expansions required to guarantee optimality – optimal in terms of the computations



Effect of the Heuristic Function

- A* Search: expands states in the order of $f = g+h$ values
- Dijkstra's: expands states in the order of $f = g$ values (pretty much)

- Intuitively: f(s) – estimate of the cost of a least cost path from start to goal via s

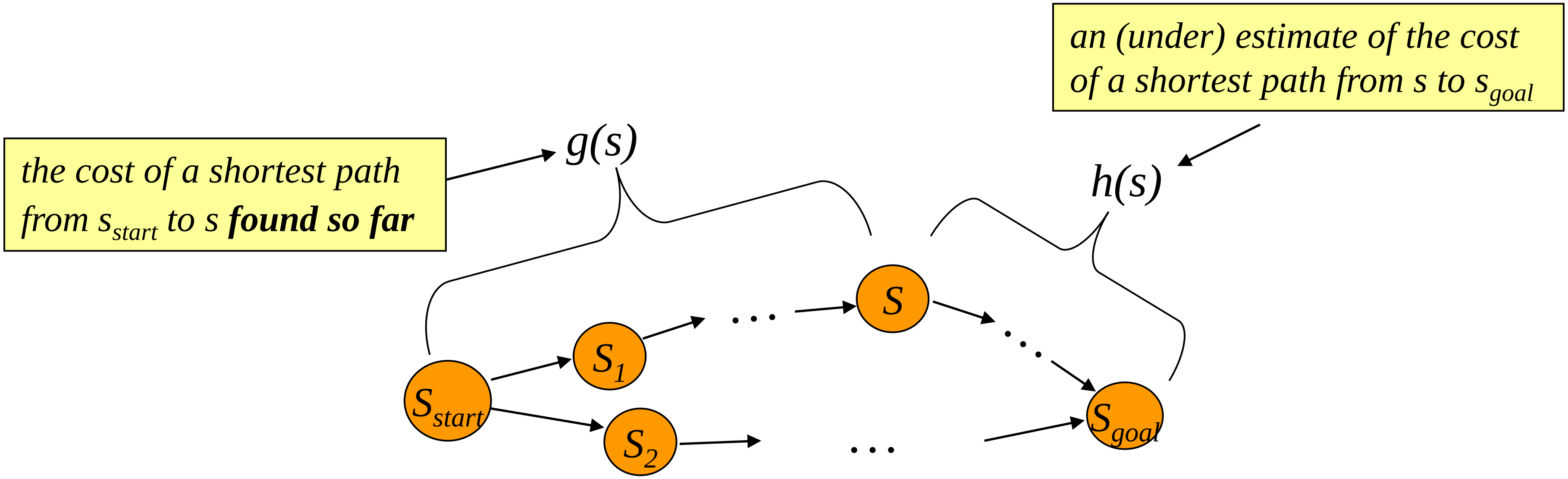


the cost of a shortest path from s_{start} to s ***found so far***

an (under) estimate of the cost of a shortest path from s to s_{goal}

Effect of the Heuristic Function

- A* Search: expands states in the order of $f = g+h$ values
- Dijkstra's: expands states in the order of $f = g$ values (pretty much)
- **Weighted A*:** expands states in the order of $f = g+\varepsilon h$ values, $\varepsilon > 1$ = bias towards states that are closer to goal



an (under) estimate of the cost of a shortest path from s to s_{goal}

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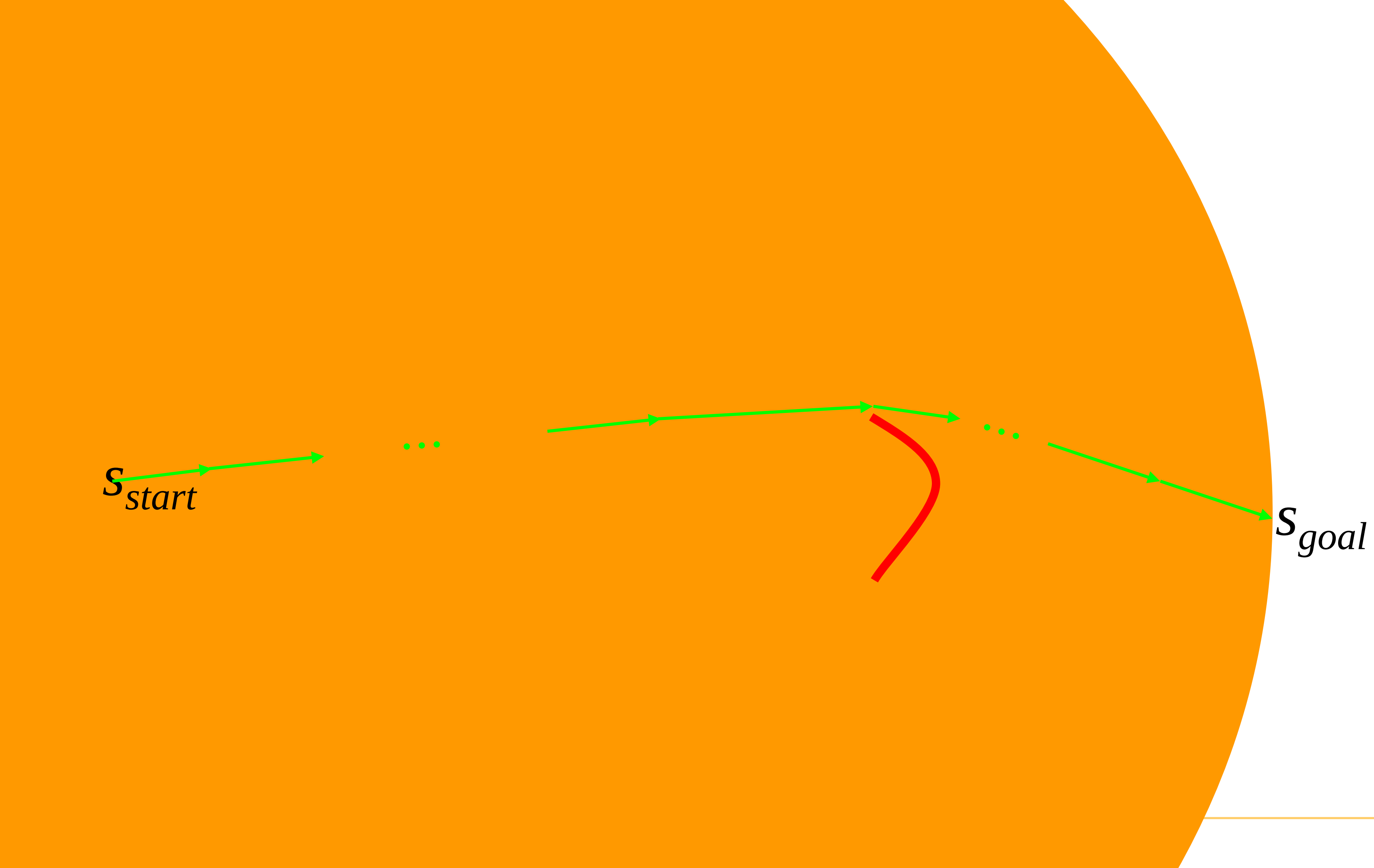
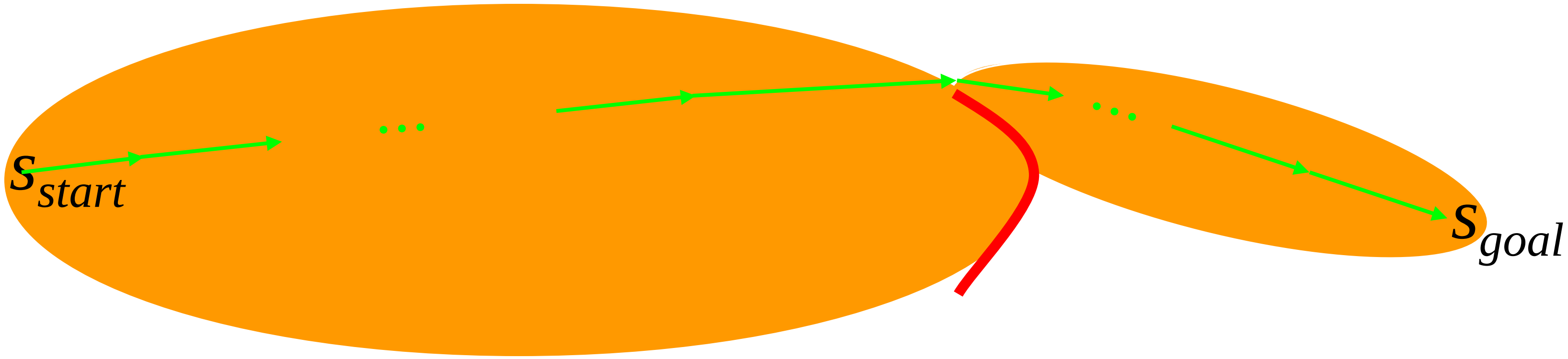
Effect of the Heuristic Function

- Dijkstra’s: expands states in the order of *f* = *g* values



Effect of the Heuristic Function

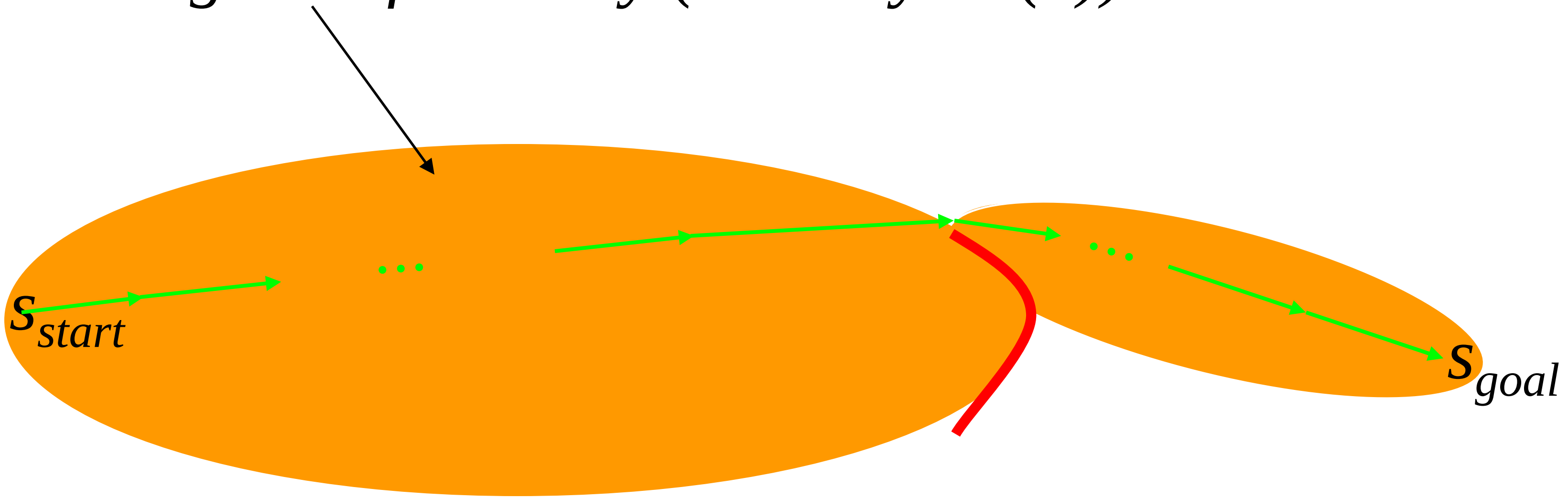
- A* Search: expands states in the order of *f* = *g*+*h* values



Effect of the Heuristic Function

- A* Search: expands states in the order of $f = g+h$ values

for large problems this results in A quickly running out of memory (memory: O(n))*



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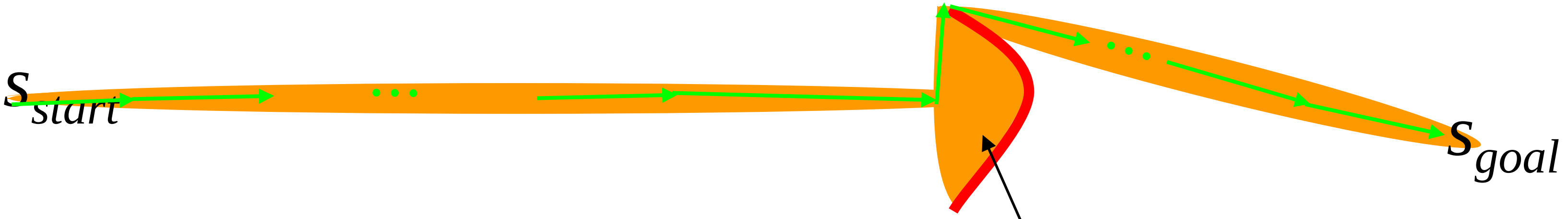

key to finding solution fast:
shallow minima for h(s)-h(s) function*

Effect of the Heuristic Function

- Weighted A* Search:
  - trades off optimality for speed
  - ε-suboptimal:

    $cost(solution) \leq \varepsilon \cdot cost(optimal\ solution)$
  - in many domains, it has been shown to be orders of magnitude faster than A*
  - research becomes to develop a heuristic function that has shallow local minima

Effect of the Heuristic Function

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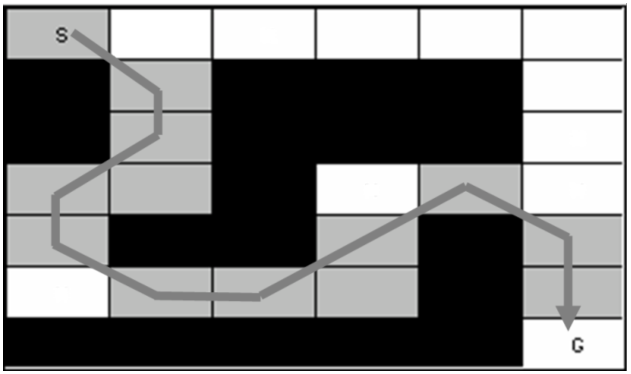
Effect of the Heuristic Function

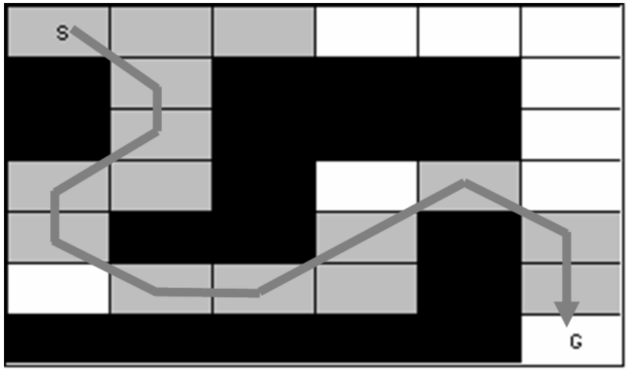
- Constructing anytime search based on weighted A*:
  - find the best path possible given some amount of time for planning
  - do it by running a series of weighted A* searches with decreasing ε:

$\varepsilon = 2.5$



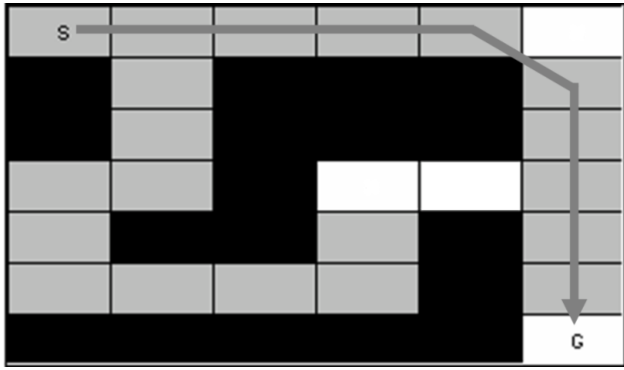
13 expansions
solution=11 moves

$\varepsilon = 1.5$



15 expansions
solution=11 moves

$\varepsilon = 1.0$



20 expansions
solution=10 moves

Effect of the Heuristic Function

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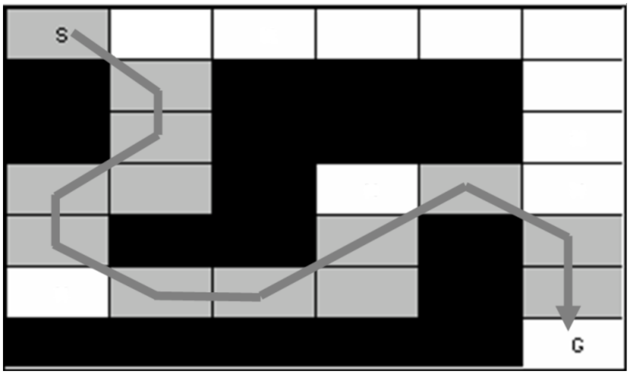

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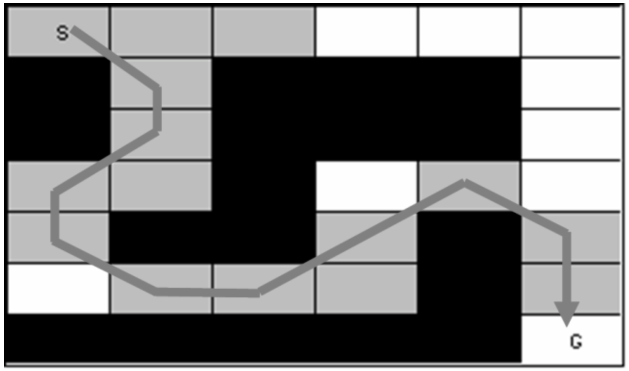

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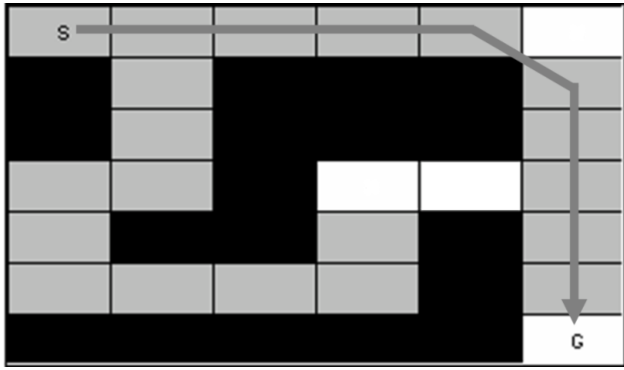

20 expansions
solution=10 moves

- Inefficient because
  - –many state values remain the same between search iterations
  - –we should be able to reuse the results of previous searches

Effect of the Heuristic Function

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  - find the best path possible given some amount of time for planning
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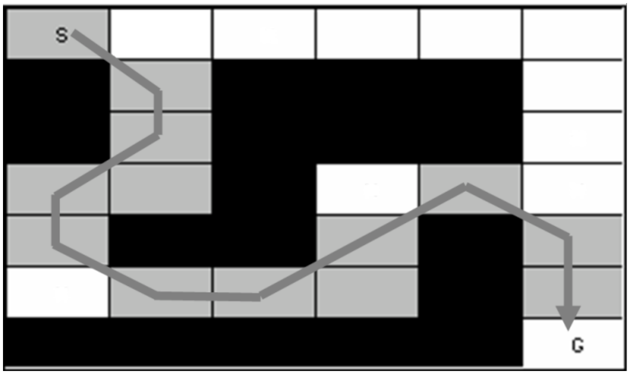
ε =2.5



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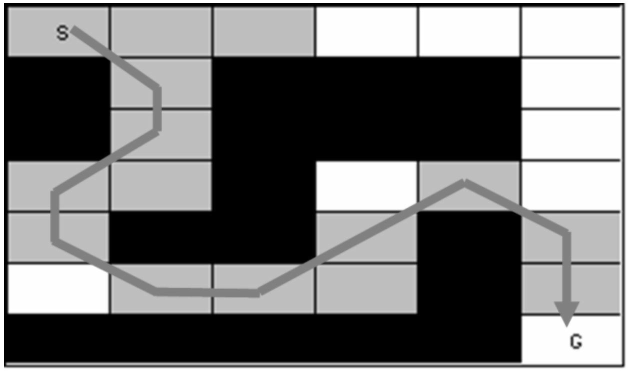

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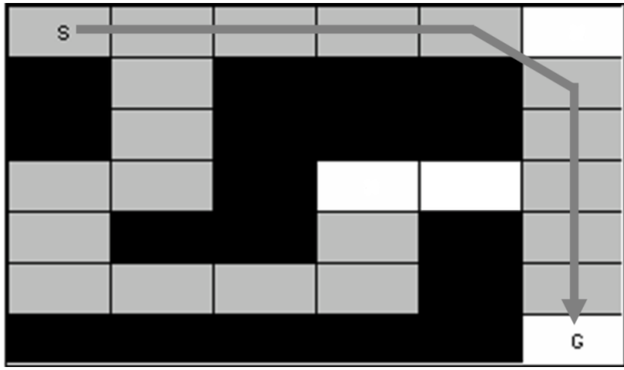


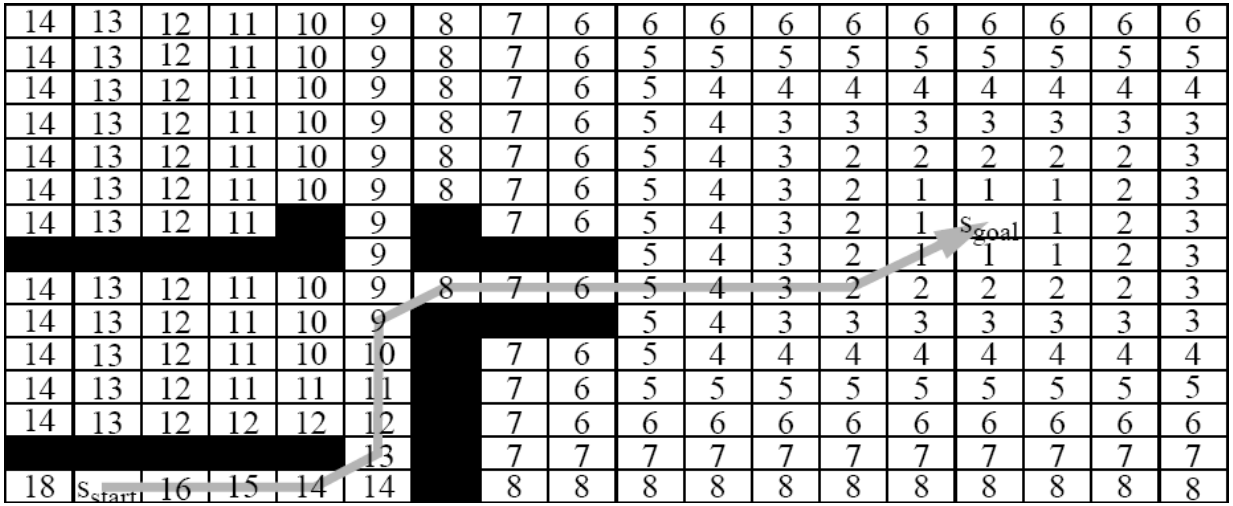
20 expansions
solution=10 moves

- ARA* [Likhachev, Gordon & Thrun, ‘04]
  - an efficient version of the above that reuses state values within any search iteration
  - uses incremental version of A*

Other Motivation for Incremental A*

- Reuse state values from previous searches

cost of least-cost paths to s_{goal} initially



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14	13	12	11	■	9	■	7	6	5	4	3	2	1	s_{goal}	1	2	3
■	■	■	■	■	9	■	■	■	5	4	3	2	1	1	1	2	3
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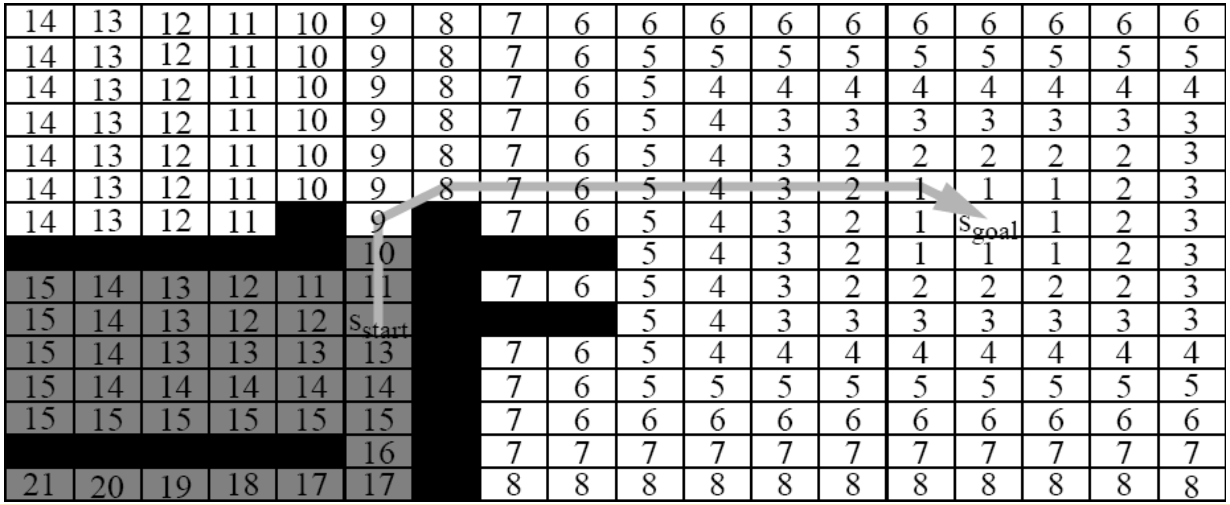
■	■	■	■	■	13	■	7	7	7	7	7	7	7	7	7	7	7
18	s_{start}	16	15	14	14	■	8	8	8	8	8	8	8	8	8	8	8

cost of least-cost paths to s_{goal} after the door turns out to be closed



14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6	6
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14	13	12	11	■	9	■	7	6	5	4	3	2	1	s_{goal}	1	2	3
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21	20	19	18	17	17	■	8	8	8	8	8	8	8	8	8	8	8

Other Motivation for Incremental A*

- Reuse state values from previous searches

cost of least-cost paths to s_{goal} initially

14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6	6
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14	13	12	11	■	9	■	7	6	5	4	3	2	1	s_{goal}	1	2	3
■	■	■	■	■	9	■	■	■	5	4	3	2	1	1	1	2	3
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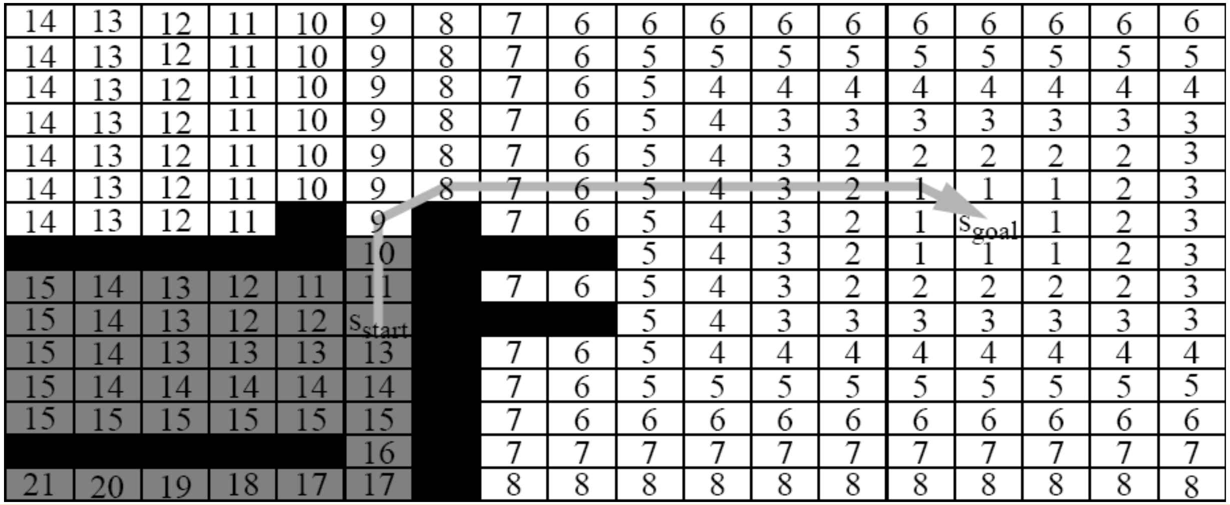
14	13	12	11	10	9	■	■	■	5	4	3	3	3	3	3	3	3
14	13	12	11	10	10	■	7	6	5								
14	13	12	11	11	11	■	7										
14	13	12	12	12	12	■	7										
■	■	■	■	■	13	■	7	7	7								
18	s_{start}	16	15	14	14	■	8	8	8	8	8	8	8	8	8	8	8

These costs are optimal g-values if search is done backwards

cost of least-cost paths to s_{goal} after the door turns out to be closed



14	13	12	11	10	9	8	7	6	6	6	6	6	6	6	6	6	6
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14	13	12	11	■	9	■	7	6	5	4	3	2	1	s_{goal}	1	2	3
■	■	■	■	■	10	■	■	■	5	4	3	2	1	1	1	2	3
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■	■	■	■	■	16	■	7	7	7	7	7	7	7	7	7	7	7
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14	13	12	11	■	9	■	7	6	5	4	3	2	1	s_{goal}	1	2	3
■	■	■	■	■	9	■	■	■	5	4	3	2	1	1	1	2	3
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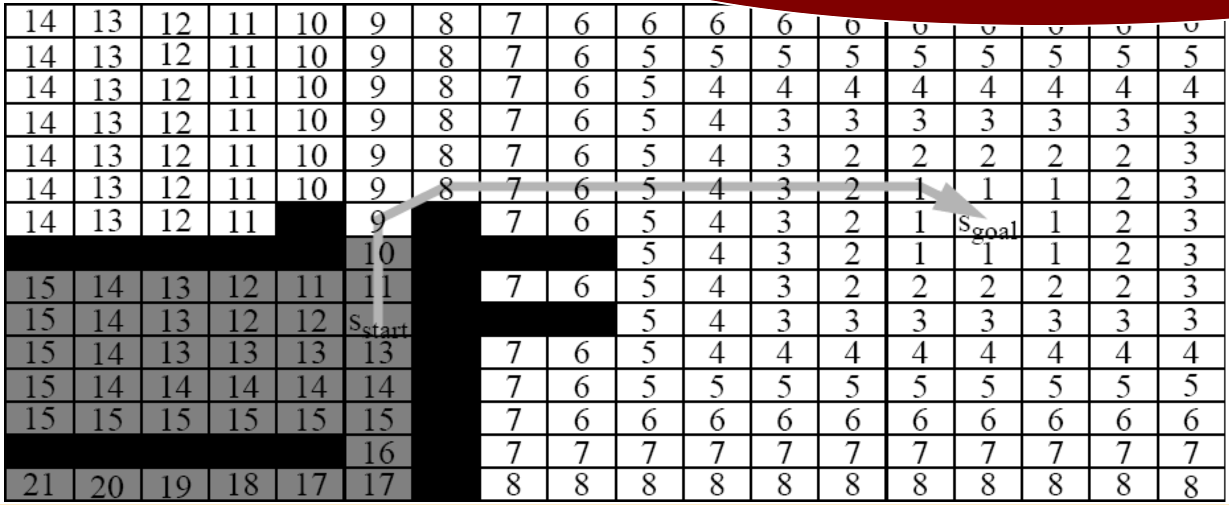
14	13	12	11	11	11	■	7	6									
14	13	12	12	12	12	■	7	6									
■	■	■	■	■	13	■	7	7	7								
18	s_{start}	16	15	14	14	■	8	8	8	8	8						

These costs are optimal g-values if search is done backwards

Can we reuse these g-values from one search to another? – incremental A*

cost of least-cost paths to s_{goal}



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14	13	12	11	■	9	■	7	6	5	4	3	2	1	s_{goal}	1	2	3
■	■	■	■	■	10	■	■	■	5	4	3	2	1	1	1	2	3
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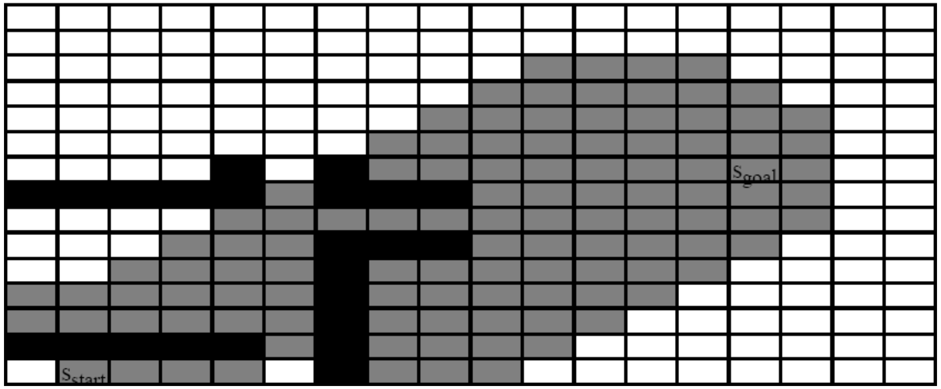
■	■	■	■	■	16	■	7	7	7	7	7	7	7	7	7	7	7
21	20	19	18	17	17	■	8	8	8	8	8	8	8	8	8	8	8

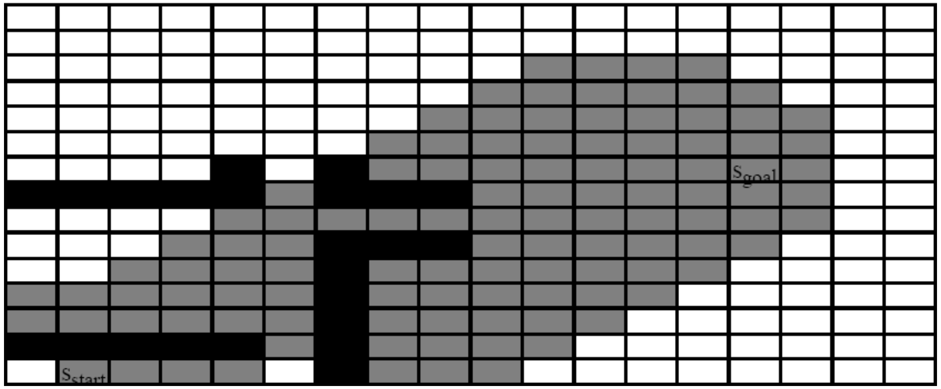
Use of Incremental A* in D* Lite [Koenig & Likhachev, ‘02]

- Reuse state values from previous searches

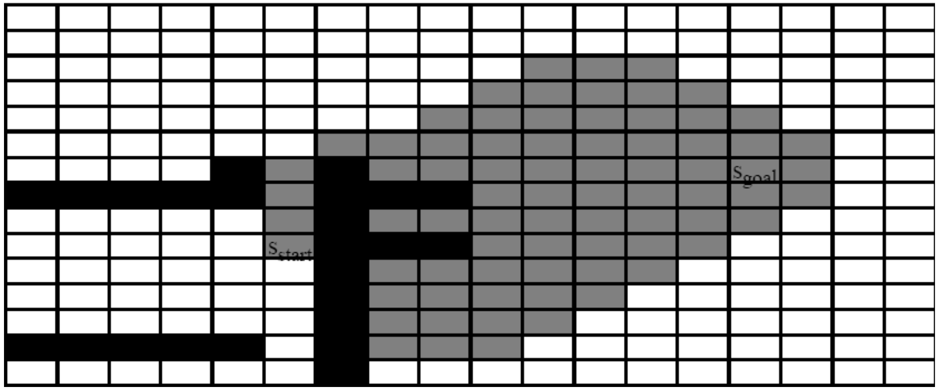
*initial search by backwards A**



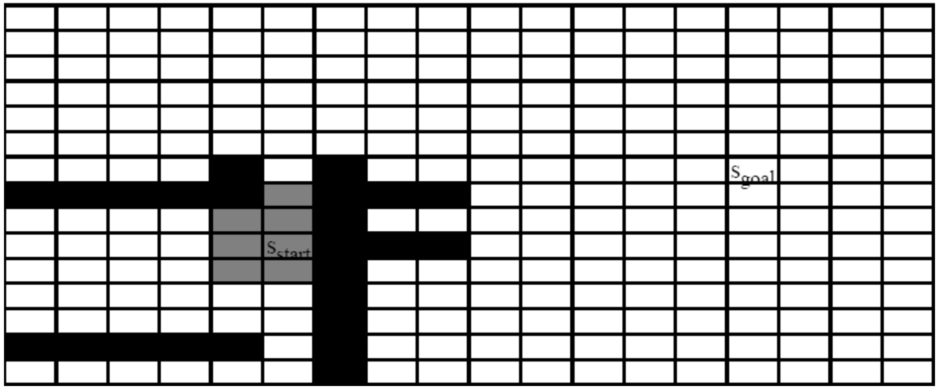
initial search by D Lite*



*second search by backwards A**



second search by D Lite*



A* with Reuse of State Values

- Alternative view of A*

```
all $v$-values initially are infinite;
ComputePath function
while($f(s_{goal}) >$ minimum $f$-value in $OPEN$ )
  remove $s$ with the smallest $[g(s)+ h(s)]$ from $OPEN$;
  insert $s$ into $CLOSED$;
  $v(s)=g(s);$
  for every successor $s'$ of $s$
    if $g(s') > g(s) + c(s,s')$
      $g(s') = g(s) + c(s,s');$
      insert $s'$ into $OPEN$;
```

A* with Reuse of State Values

- Alternative view of A*

v-value – the value of a state during its expansion (infinite if state was never expanded)

```
all $v$-values initially are infinite;
**ComputePath function**
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```

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  $v(s)=g(s)$;
  for every successor s' of s
      if $g(s') > g(s) + c(s,s')$
        $g(s') = g(s) + c(s,s')$;
        insert s' into OPEN;
```

- $g(s') = \min_{s'' \in pred(s')} v(s'') + c(s'',s')$

A* with Reuse of State Values

- Alternative view of A*

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```

- $g(s') = \min_{s'' \in pred(s')} v(s'') + c(s'',s')$
- *OPEN*: a set of states with $v(s) > g(s)$

  all other states have $v(s) = g(s)$

overconsistent state

consistent state

A* with Reuse of State Values

- Alternative view of A*

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ComputePath function
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    if $g(s') > g(s) + c(s,s')$
      $g(s') = g(s) + c(s,s');$
      insert s' into OPEN;
```

- $g(s') = \min_{s'' \in pred(s')} v(s'') + c(s'',s')$
- *OPEN:* a set of states with $v(s) > g(s)$
  all other states have $v(s) = g(s)$
- this A* expands overconsistent states in the order of their f-values

A* with Reuse of State Values

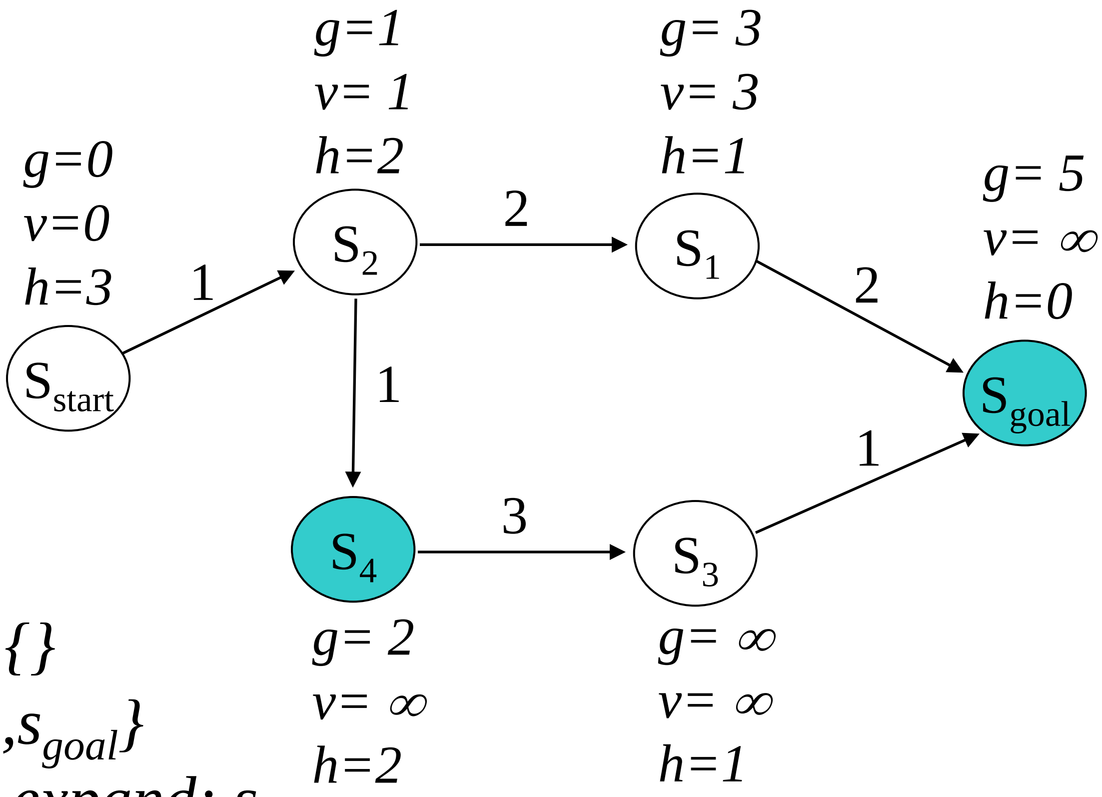
- Making A* reuse old values:

```
initialize OPEN with all overconsistent states;
ComputePathwithReuse function
while($f(s_{goal})$ > minimum $f$-value in $OPEN$ )
   remove $s$ with the smallest $[g(s)+ h(s)]$ from $OPEN$;
   insert $s$ into $CLOSED$;
   $v(s)=g(s)$;
   for every successor $s'$ of $s$
      if $g(s') > g(s) + c(s,s')$
         $g(s') = g(s) + c(s,s')$;
         insert $s'$ into $OPEN$;
```

all you need to do to make it reuse old values!

- $g(s') = \min_{s'' \in pred(s')} v(s'') + c(s'',s')$
- *OPEN:* a set of states with $v(s) > g(s)$
  all other states have $v(s) = g(s)$
- this A* expands overconsistent states in the order of their f-values

A* with Reuse of State Values



CLOSED = {}

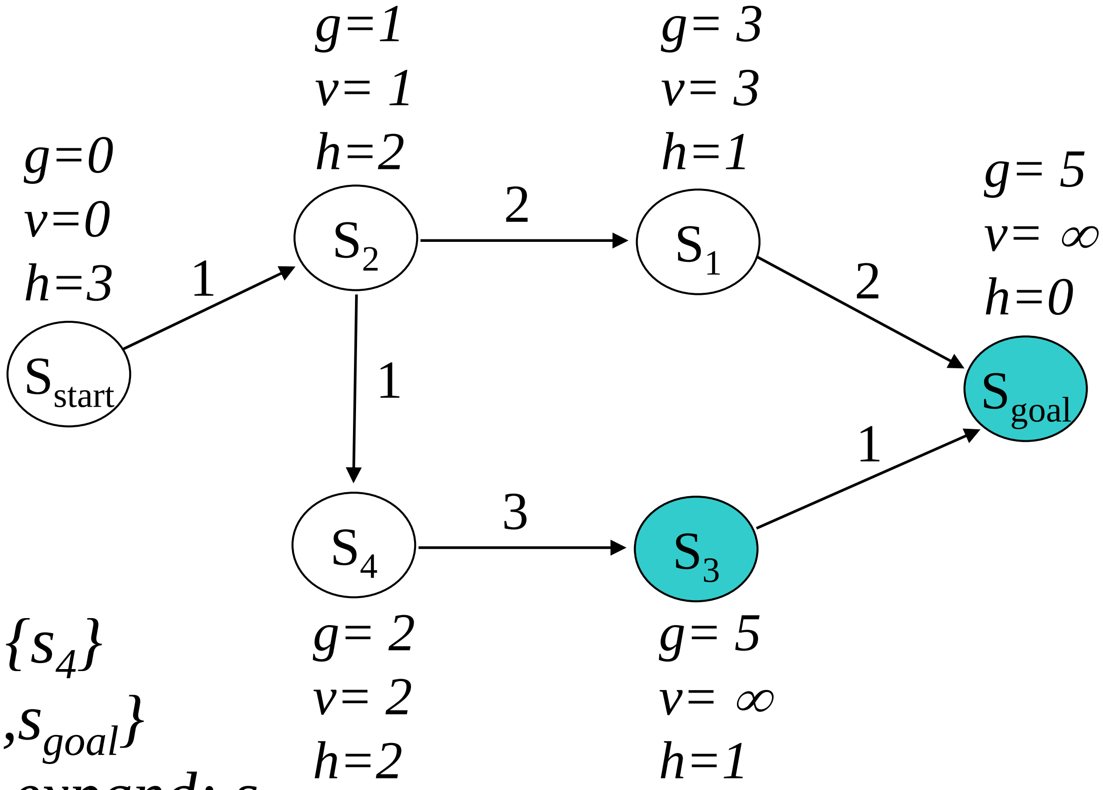
OPEN = {s_4,s_{goal}}

next state to expand: s_4

$g(s') = \min_{s'' \in pred(s')} v(s'') + c(s'',s')$

initially OPEN contains all overconsistent states

A* with Reuse of State Values

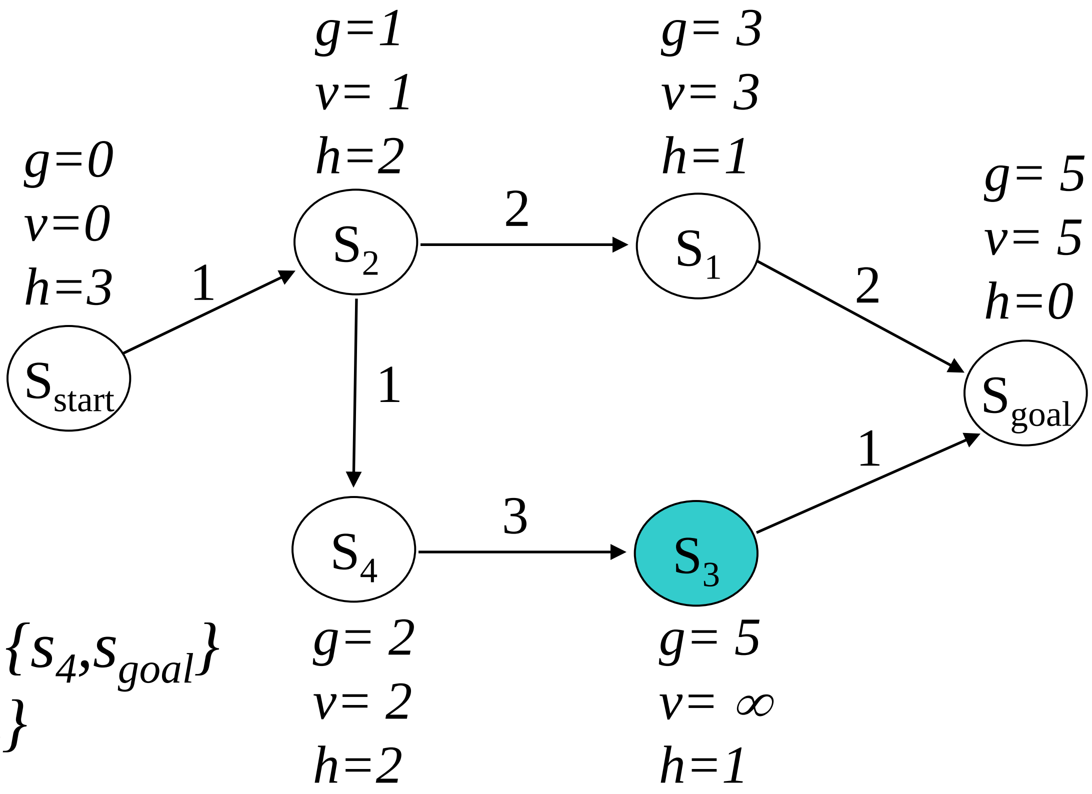


CLOSED = $\{s_4\}$

OPEN = $\{s_3, s_{goal}\}$

next state to expand: s_{goal}

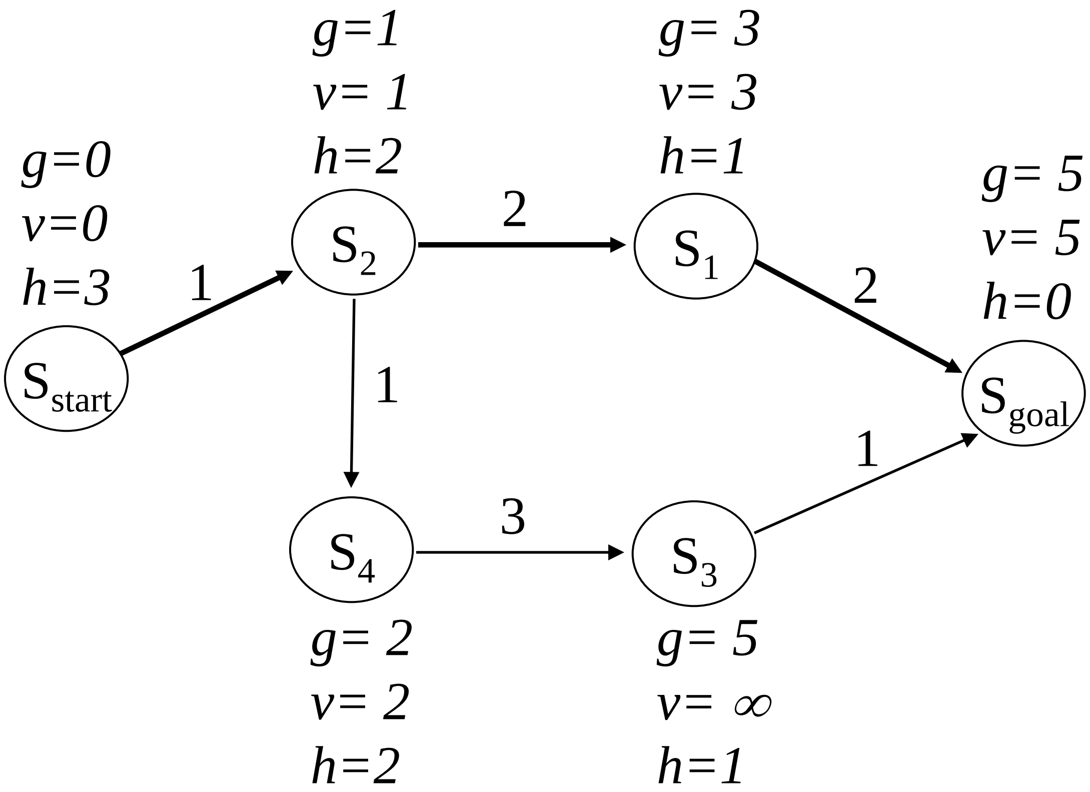
A* with Reuse of State Values



CLOSED = {s_4,s_{goal}}
OPEN = {s_3}
done

after ComputePathwithReuse terminates:
all g-values of states are equal to final A g-values*

A* with Reuse of State Values



we can now compute a least-cost path

A* with Reuse of State Values

- Making weighted A* reuse old values:

```
initialize OPEN with all overconsistent states;
ComputePathwithReuse function
while($f(s_{goal}) >$ minimum $f$-value in OPEN )
    remove $s$ with the smallest $[g(s)+ \varepsilon h(s)]$ from OPEN;
    insert $s$ into CLOSED;
    $v(s)=g(s)$;
    for every successor $s'$ of $s$
        if $g(s') > g(s) + c(s,s')$
            $g(s') = g(s) + c(s,s')$;
            if $s'$ not in CLOSED then insert $s'$ into OPEN;
```

just make sure no state is expanded multiple times

Anytime Repairing A* (ARA*)

- Efficient series of weighted A* searches with decreasing ε:

```
set $\varepsilon$ to large value;
$g(s_{start}) = 0$; $v$-values of all states are set to infinity; $OPEN = \{s_{start}\}$;
while $\varepsilon \geq 1$
    $CLOSED = \{\}$;
    ComputePathwithReuse();
    publish current $\varepsilon$ suboptimal solution;
    decrease $\varepsilon$;
    initialize $OPEN$ with all overconsistent states;
```

ARA*

- Efficient series of weighted A* searches with decreasing ε:

```
set $\varepsilon$ to large value;
$g(s_{start}) = 0$; $v$-values of all states are set to infinity; $OPEN = \{s_{start}\}$;
while $\varepsilon \geq 1$
    $CLOSED = \{\}$;
    ComputePathwithReuse();
    publish current $\varepsilon$ suboptimal solution;
    decrease $\varepsilon$;
    initialize $OPEN$ with all overconsistent states;
```

need to keep track of those

ARA*

- Efficient series of weighted A* searches with decreasing ε:

```
initialize OPEN with all overconsistent states;
ComputePathwithReuse function
while($f(s_{goal})$ > minimum f-value in OPEN )
    remove s with the smallest $[g(s)+ \varepsilon h(s)]$ from OPEN;
    insert s into CLOSED;
    $v(s)=g(s);$
    for every successor s' of s
        if $g(s') > g(s) + c(s,s')$
            $g(s') = g(s) + c(s,s');$
            if s' not in CLOSED then insert s' into OPEN;
            otherwise insert s' into INCONS
```

- *OPEN U INCONS* = all overconsistent states

ARA*

- Efficient series of weighted A* searches with decreasing ε:

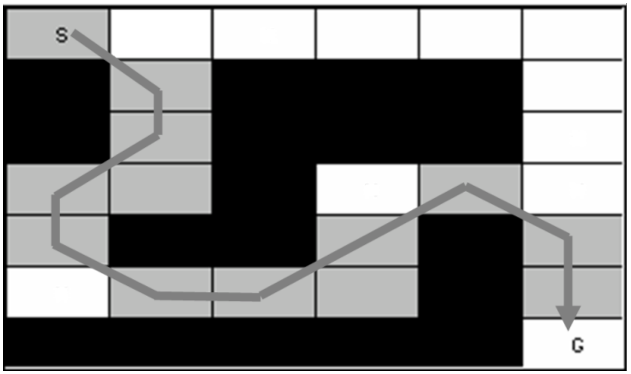
```
set $\varepsilon$ to large value;
$g(s_{start}) = 0$; $v$-values of all states are set to infinity; $OPEN = \{s_{start}\}$;
while $\varepsilon \geq 1$
    $CLOSED = \{\}$; $INCONS = \{\}$;
    ComputePathwithReuse();
    publish current $\varepsilon$ suboptimal solution;
    decrease $\varepsilon$;
    initialize $OPEN = OPEN \cup INCONS$;
```

all overconsistent states (exactly what we need!)

ARA*

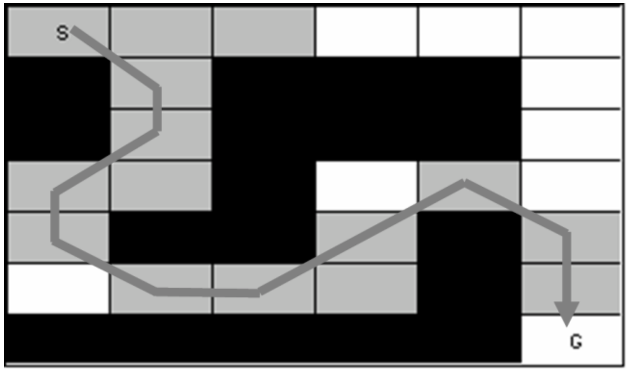
- **A series of weighted A* searches**

ε =2.5



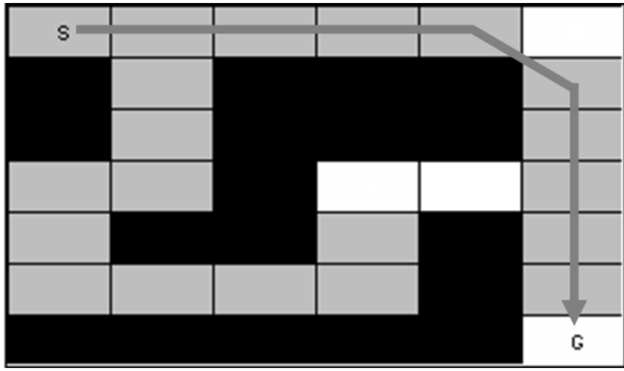
13 expansions
solution=11 moves

ε =1.5



15 expansions
solution=11 moves

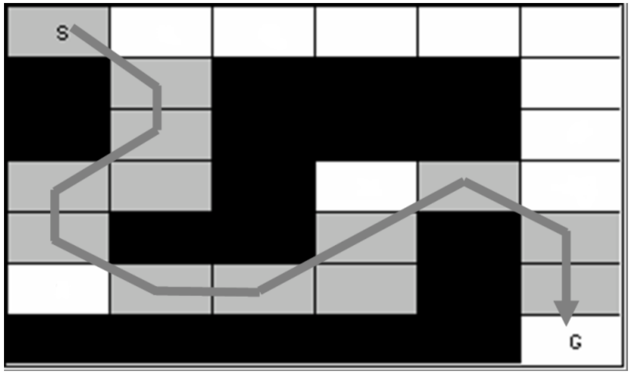
ε =1.0



20 expansions
solution=10 moves

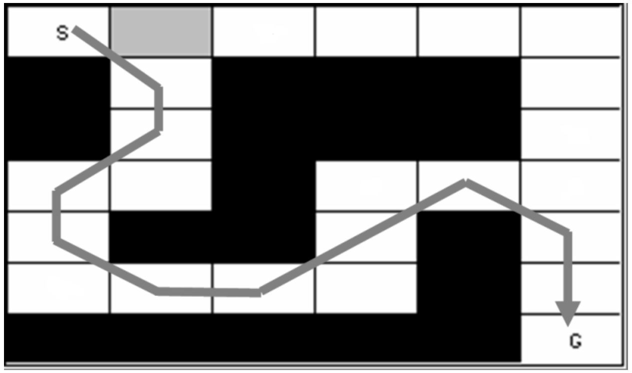
- **ARA***

ε =2.5



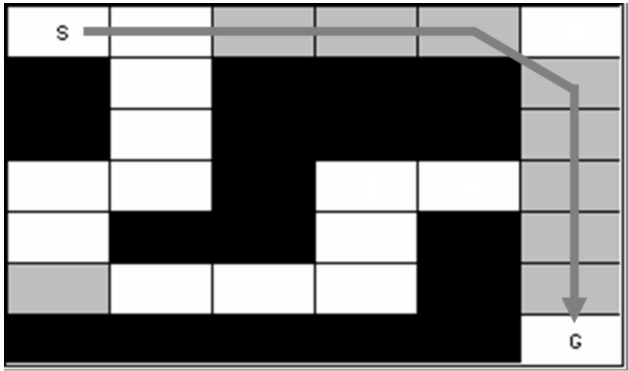
13 expansions
solution=11 moves

ε =1.5



1 expansion
solution=11 moves

ε =1.0



9 expansions
solution=10 moves

What I will talk about

- Graph representations (implemented as environments for SBPL)
  - 3D (x,y,θ) lattice-based graph (within SBPL)
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  - Cart planning (separate SBPL-based package)
  - Lattice-based arm motion graph (separate SBPL-based motion planning module)
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- Graph searches (implemented within SBPL)
  - ARA* - anytime version of A*
  - Anytime D* - anytime incremental version of A*
  - R* - a randomized version of A* (will not talk about)

- Heuristic functions (implemented as part of environments)

- Overview of how SBPL code is structured

- What's coming

Anytime and Incremental Planning

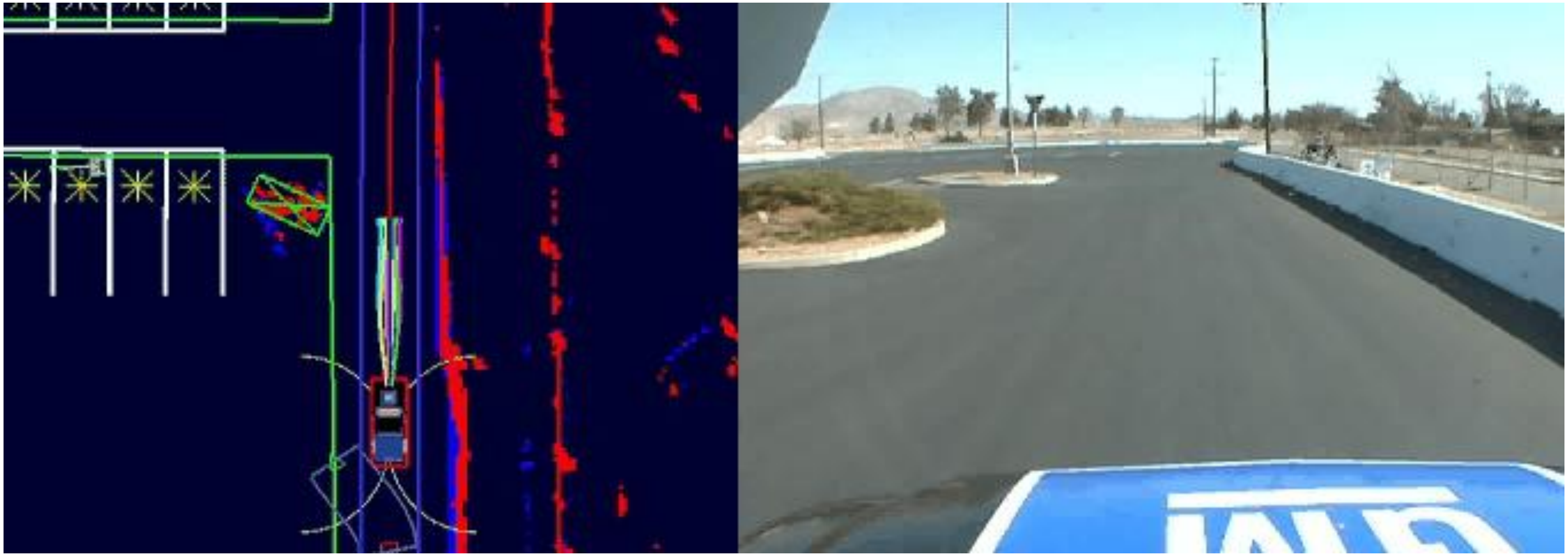
- Anytime D* [Likhachev et al., ‘2008]:
  - decrease ε and update edge costs at the same time
  - re-compute a path by reusing previous state-values

```
set $\varepsilon$ to large value;
until goal is reached
    ComputePathwithReuse();    //modified to handle cost increases
    publish $\varepsilon$-suboptimal path;
    follow the path until map is updated with new sensor information;
    update the corresponding edge costs;
    set $s_{start}$ to the current state of the agent;
    if significant changes were observed
        increase $\varepsilon$ or replan from scratch;
    else
        decrease $\varepsilon$;
```

Anytime and Incremental Planning

- Anytime D* in Urban Challenge

*planning on 4D (<x,y,orientation,velocity>) multi-resolution lattice using Anytime D**
[Likhachev & Ferguson, '09]



part of efforts by Tartanracing team from CMU for the Urban Challenge 2007 race

Other Uses of Incremental A*

- Whenever planning is a repeated process:
  - improving a solution (e.g., in anytime planning)
  - re-planning in dynamic and previously unknown environments
  - adaptive discretization
  - many other planning problems can be solved via iterative planning

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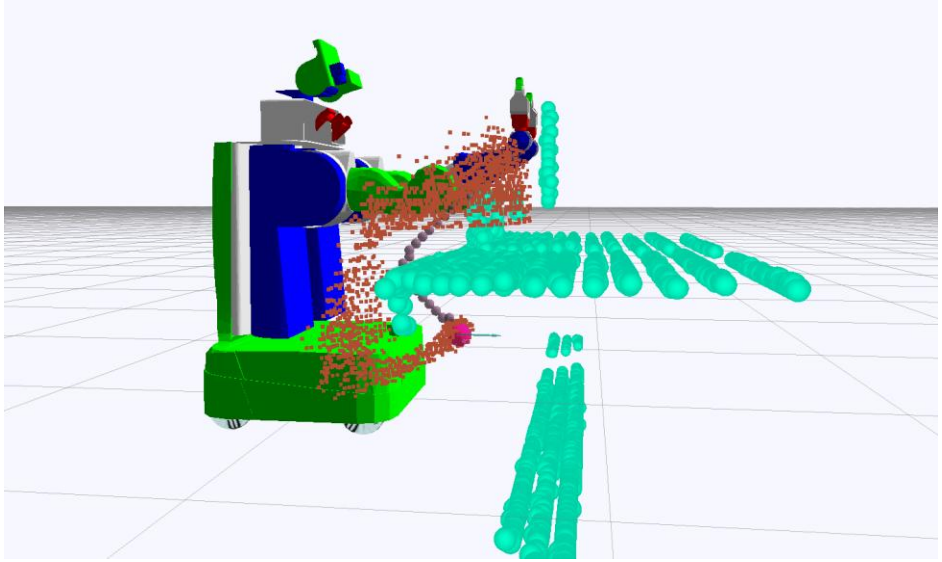
Heuristic Functions

- 2D (x,y) Dijkstra's taking into account all obstacles for:
  - 3D (x,y,θ) lattice-based graph
  - 3D (x,y,θ) lattice-based graph for 3D (x,y,z) spaces
  - cart planning

- Angle distance to the fully open door for:
  - door opening planning

- 3D (x,y,z) Dijkstra's for the end-effector taking into account all obstacles for:
  - lattice-based arm motion graph (separate SBPL-based motion planning module)



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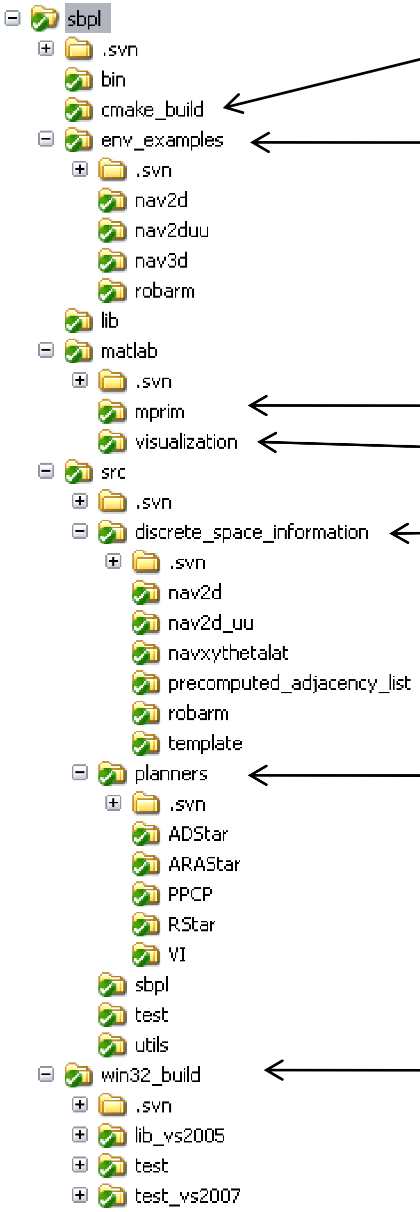
Structure of SBPL



- sbpl
  - .svn
  - bin
  - cmake_build
  - env_examples
    - .svn
    - nav2d
    - nav2duu
    - nav3d
    - robarm
  - lib
  - matlab
    - .svn
    - mprim
    - visualization
  - src
    - .svn
    - discrete_space_information
      - .svn
      - nav2d
      - nav2d_uu
      - navxythetalat
      - precomputed_adjacency_list
      - robarm
      - template
    - planners
      - .svn
      - ADStar
      - ARAStar
      - PPCP
      - RStar
      - VI
    - sbpl
    - test
    - utils
  - win32_build
    - .svn
    - lib_vs2005
    - test
    - test_vs2007

For compiling under Linux/Windows using CMake

Example configuration files to run main.cpp on. These can be used to test planners outside of ROS, just running compiled test program in bin directory from the command line

Motion primitives used as input for 3D (x,y,θ) lattice-based planning and matlab files to generate new motion primitives

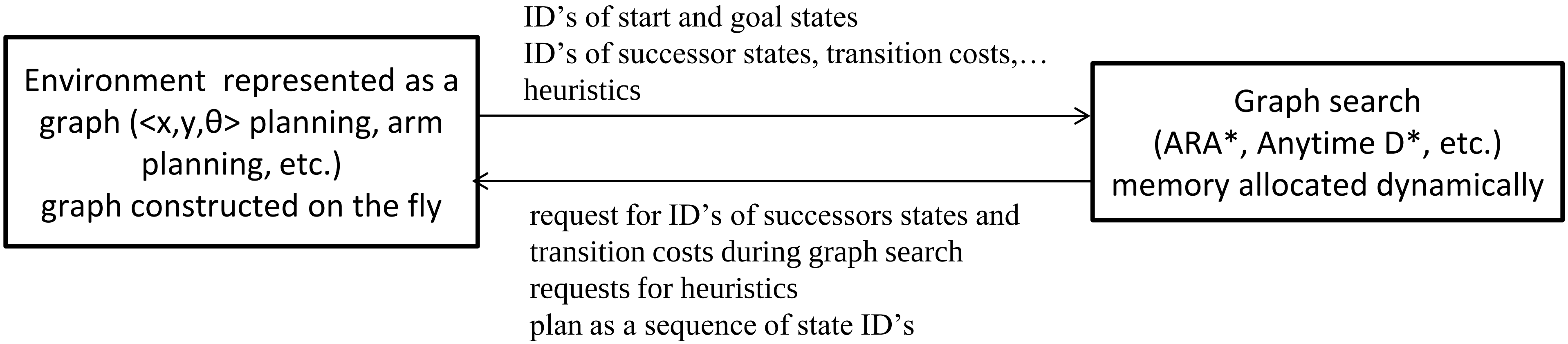
Few matlab scripts to visualize 3D paths and motion primitives

Environments (.cpp and .h files defining planning problems as graphs)

Domain-independent graph searches

For compiling under Windows using Visual Studio

Structure of SBPL



Environment represented as a graph (<x,y,θ> planning, arm planning, etc.)
graph constructed on the fly

ID’s of start and goal states
ID’s of successor states, transition costs,…
heuristics

Graph search
(ARA*, Anytime D*, etc.)
memory allocated dynamically

request for ID’s of successors states and transition costs during graph search
requests for heuristics
plan as a sequence of state ID’s

Structure of SBPL

- Look at Main.cpp for examples for how to use SBPL:

```
EnvironmentNAVXYTHETALAT environment_navxythetalat;
if(!environment_navxythetalat.InitializeEnv(argv[1], perimeterptsV, NULL))
{
            SBPL_ERROR("ERROR: InitializeEnv failed\n");
            throw new SBPL_Exception();
}
if(!environment_navxythetalat.InitializeMDPCfg(&MDPCfg))
{
            SBPL_ERROR("ERROR: InitializeMDPCfg failed\n");
            throw new SBPL_Exception();
}
//plan a path
vector<int> solution_stateIDs_V;
bool bforwardsearch = false;
ADPlanner planner(&environment_navxythetalat, bforwardsearch);
if(planner.set_start(MDPCfg.startstateid) == 0)
{
            SBPL_ERROR("ERROR: failed to set start state\n");
            throw new SBPL_Exception();
 }
 if(planner.set_goal(MDPCfg.goalstateid) == 0)
  {
            SBPL_ERROR("ERROR: failed to set goal state\n");
            throw new SBPL_Exception();
  }
  planner.set_initialsolution_eps(3.0);

 bRet = planner.replan(allocated_time_secs, &solution_stateIDs_V);
 SBPL_PRINTF("size of solution=%d\n",(unsigned int)solution_stateIDs_V.size());
```

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What's coming

- Planning in Dynamic Environments
- Planning for Spring-loaded Doors
- ROS package for (x,y,θ) planning while accounting for the whole body of PR2 in 3D (x,y,z)

http://www.ros.org/wiki/sbpl

Thanks to Willow Garage for the support of SBPL!